Managing Longevity Risk with Longevity Bonds

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“...the average duration of human life is proved to have increased in recent years. The calculations of various life assurance and annuity offices, among other figures which cannot go wrong, have established the fact … it is governed by the laws that govern lives in the aggregate.”

– Charles Dickens, Hard Times (1854)
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1 Introduction

The average life expectancy has steadily grown over the last decades, particularly in developed countries. Due to improved standards of living and ongoing advances in medicine, people are living longer: the main factor driving up life expectancy has been the reduction of death rates among the elderly (Denuit et al., 2007). This positive development, together with the stochastic nature of mortality, poses a challenge to pension plans and life insurers, many of whom have based the pricing of their products on mortality forecasts that now appear to have underestimated the realized reductions in mortality. As an example, Figure 1 shows that the life expectancy estimates for insured males over 60 in the UK have been systematically too low. Different lines, labelled in the figure by the year of projection, depict the projected life expectancies in years (vertical axis) for males aged 60 at a given future year (horizontal axis). As mortality rates have decreased more than anticipated, insurance companies are left paying out annuity payments longer than expected, bearing the costs of greater longevity.

The calculation of expected present values for pricing and liability considerations requires an accurate mortality projection to avoid underestimation of future costs.
Pension and life insurance companies have generally used deterministic mortality intensities for these purposes. These mortality intensities are usually estimated using historical data from the insurance policies comprising the portfolios of the companies. Taking the estimated values as given, the companies have traditionally focused on minimizing the unsystematic mortality risk, that is, the risk associated with the randomness of deaths in an insurance portfolio with known mortality (Dahl et al., 2007). According to the law of large numbers, the unsystematic mortality risk can be diversified away by holding a large enough portfolio.

However, it is now widely acknowledged that mortality is a stochastic process (e.g. Cairns et al., 2006a), which means that insurance companies face systematic mortality risk in addition to unsystematic risk. The systematic mortality risk is the risk associated with changes in the underlying mortality intensity, and it is not diversifiable\(^1\).

Insurance companies have traditionally had rather limited means to hedge systematic mortality-related risks. This is now changing, with last years seen an increasing number of insurance-related securitizations. These have involved the transfer of some or all of the underlying risks from the insurers to the capital markets, whose capacity to assume risk is far greater than any insurer’s. In addition to providing insurers with a tool to hedge mortality risks, these securitizations offer investors means to diversify their portfolios, as insurance-linked securities have had low or no correlation with financial markets (Cowley and Cummins, 2005).

In this paper, we examine a class of securities proposed for managing longevity risk, longevity bonds. These are bonds whose coupon payments are linked to the proportion of a reference population surviving to certain ages. The bonds provide a hedge against longevity risk: if the insured live longer than anticipated, the insurance companies have to make annuity payments longer, but receive higher coupons from their bond positions, offsetting the losses incurred on insurance portfolios.

Our purpose is to provide an overview of and introduction to mortality risk management using risk transfer securitizations, particularly longevity bonds. We briefly discuss the hedging of longevity risk with these bonds and outline issues related to their use as

\(^1\) See e.g. Milevsky et al. (2006) for an overview of how the law of large numbers breaks down when pricing mortality-linked claims under stochastic (as opposed to deterministic) mortality intensities.
hedging instruments. In order to price and value mortality derivative securities, one has to combine financial contingent claims pricing theory with mortality modelling. We adopt a stochastic mortality model proposed by Cairns, Blake and Dowd (2006) to forecast future mortality development. This model is calibrated to Finnish mortality data and used to derive prices for longevity bonds linked to the mortality experience of Finnish population. As an illustration, we compare the prices obtained from Monte Carlo simulations with those obtained from deterministic projections used within Finnish life insurance industry.

The structure of the paper is as follows. We start in Chapter 2 by introducing notation and definitions used in the remainder of the paper. Chapter 3 discusses the concept of securitization and outlines mortality-linked transactions carried out so far, together with the outlook of the market. Chapter 4 considers the characteristics and pricing of longevity bonds, as well as practical issues related to hedging longevity risk using these securities. In Chapter 5 we introduce the CBD-model used for mortality forecasting. The model calibration and simulation results are presented in Chapter 6. Chapter 7 concludes.

2 Notation and Definitions

2.1 General Notation

National mortality data are typically published on an annual basis, and it is therefore natural to adopt a discrete-time framework for modelling mortality. We define calendar year \( t \) as the time period running from \( t \) to \( t + 1 \). Let \( m(t, x) \) be the crude or actual death rate\(^2\) for age \( x \) in calendar year \( t \),

\[
m(t, x) = \frac{D(t, x)}{E(t, x)} = \frac{\text{#deaths during calendar year } t \text{ aged } x}{\text{average population during calendar year } t \text{ aged } x}.
\]

The average population, or the exposure, is usually based on the estimate of the population aged \( x \) last birthday in the middle of a calendar year, or on the average of

\(^2\) We will write \( m(t, x) \) for the crude death rate and \( \hat{m}(t, x) \) for the underlying or expected death rate.
population estimates at the beginning and end of a year (Cairns et al., 2008a).

A second measure of mortality is the mortality rate \( q(t,x) \), the underlying probability that an individual aged exactly \( x \) at time \( t \) will die before reaching age \( x + 1 \), that is, between \( t \) and \( t + 1 \). If we denote by \( T(t,x) \) the remaining lifetime of such an individual, the mortality rate can be written as \( q(t,x) = \Pr[T(t,x) \leq 1] \). \( q(t,x) \) is defined for integer values of \( x \) and \( t \), and is observable only after time \( t + 1 \). The complement of mortality rate, \( p(t,x) = 1 - q(t,x) \), is the probability that an individual aged \( x \) in calendar year \( t \) will survive until age \( x + 1 \), that is, \( p(t,x) = \Pr[T(t,x) > 1] \).

We introduce one more mortality-related measure, the force of mortality \( \mu(t,x) \). This is the instantaneous death rate for individuals aged \( x \) at time \( t \). This means that for small intervals of time, \( dt \), the probability of death between \( t \) and \( t + dt \) is approximately \( \mu(t,x)dt \) .

The relationship between mortality rate \( q(t,x) \) and force of mortality \( \mu(t,x) \) can be written as

\[
q(t,x) = 1 - \exp\left[ -\int_{t}^{t+1} \mu(u,x-t+u)du \right] \tag{1}
\]

Given our discrete-time framework, we make the following simplifying assumption (see, e.g. Brouhns et al., 2002, and Denuit et al., 2007):

\[
\mu(t+s,x+u) = \mu(t,x) \quad \forall \ s, u \in [0,1), \tag{2}
\]

that is, the force of mortality stays constant over each integer age \( x \) and calendar year \( t \). Given this assumption of piecewise constant forces of mortality, we have

\[
q(t,x) = 1 - \exp[-\mu(t,x)] \tag{3}
\]

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\[3\] Put another way, \( \mu(x,t) = \lim_{dt \to 0} \Pr\left[ x < T(0,t-x) \leq x + dt \mid T(0,t-x) > x \right] \].
2.2 Types of Mortality-Related Risk

Following Cairns et al. (2006b), we use the term *mortality risk* in this paper to encompass all mortality-related risks that an insurance company faces. This includes *longevity risk* (the risk of the insured living longer than expected) as well as *short term, catastrophic mortality risk* (the risk that mortality rates are much greater than expected over short periods of time). More specifically, longevity risk can be defined as the risk of aggregate mortality of a specified reference population deviating from its expected value (at given age and over specified time horizon).

The uncertainty in future mortality can be divided into two components: systematic and unsystematic. The unsystematic mortality risk arises from the fact that, even if the true mortality rate is known, the actual number of deaths $D(t,x)$ will be random *a priori*. The systematic mortality risk is related to unexpected changes in the underlying mortality rate, and affects all lives in a portfolio in the same way. (Dahl et al., 2007)

The division of mortality risk into systematic and unsystematic components is analogous to that used in financial markets literature. We make the common assumption that investors do not require a risk premium to hold unsystematic risk, as this can be diversified away by holding a large enough portfolio, under the usual assumption that individual lifetimes are independent random variables. In contrast, systematic mortality risk is not diversifiable, and leads to the incorporation of a risk premium.

3 Securization of Longevity Risk

Exposure to longevity risk is a serious concern for insurance companies, but the means to manage it have been scarce. Traditional methods for managing longevity risk have focused on participating policies, and financing through capital reserves (Wills and Sherris, 2008). Reinsurance is another possible solution, but it can often be very expensive to acquire, as reinsurers have been reluctant to accept longevity risk\(^4\) due to

\(^4\) According to Richards and Jones (2004), reinsurers wouldn’t take longevity risk, as a rule, unless it was: 1) for an existing client, 2) capped to a relatively small level, and 3) part of an overall package of risks in addition to longevity.
its systematic (undiversifiable) nature, some describing it as “toxic” (Cowley and Cummins, 2005, and Wills and Sherris, 2008). Insurance companies may also be reluctant to buy long-term reinsurance coverage because of exposure to credit risk (Denuit et al., 2007). Finally, if an insurer runs both life insurance and annuity businesses, it may be able to utilize natural hedging to some extent, as the values of life insurance and annuity liabilities tend to move in opposite directions in response to a change in the underlying mortality rates (see Cox and Lin, 2005, for an overview).

Securitization of longevity risk can offer the insurers an effective tool for managing mortality-related risk. It provides means to transfer illiquid risks into financial markets, and allows pooling of these risks. Mortgage and other asset-backed securities have been the main focus of securitizations (Wills and Sherris, 2008), but recent years have seen increasing interest in securitization opportunities available within insurance industry, both from the part of insurance companies and from potential investors.

3.1 Overview of the Securitization Process

In the securitization process, a pool of assets or rights to a set of cash flows are unbundled and then repackaged into securities that are traded in capital markets. Almost any cash flow can (at least in principle) be candidate for securitization. The transaction begins with an originator (e.g., a life insurance company) providing a product (e.g., insurance policy or annuity) to client, who in turn agrees to make a series of payments to the provider. The asset generated by the sale of the originator’s products (that is, the present value of the cash flows to be received) is then usually transferred to a special purpose vehicle (SPV), a passive financial vehicle whose only purpose is to house the asset. The SPV issues securities (with the asset serving as collateral) to investors, raising funds from them. The proceeds from the issuance are then transferred in part or in full to the originator. See Cowley and Cummins (2005) for an extensive overview of securitization in the life insurance industry. Figure 2 presents a simplified example of the cash flows in a mortality-linked transaction.
From an investor’s perspective, securitization can create new classes of non-redundant securities that appeal to investors with different risk preferences and enable investors to improve portfolio efficiency. As Cowley and Cummins (2005) point out, securities based on catastrophic property, mortality or longevity risk are non-redundant because the risks covered by these are not otherwise tradable in the capital markets. Securities based on these risks are also generally felt to have low correlation with systematic market risk, making them potentially attractive for diversification purposes.

In this article, we consider pure risk transfer securitizations that can be used to protect the originating insurer against mortality risk or longevity risk. These transactions allow insurers to transfer some or all of the systematic risk, that is, the risk due to the possible occurrence of a catastrophic event or adverse aggregate mortality development, off their balance sheets to the capital markets.

Several mortality-linked instruments have been proposed in the literature, such as longevity bonds, longevity or survivor swaps (see e.g., Dowd et al., 2006) and mortality derivatives (see Dawson et al., 2007). We will concentrate on longevity bonds in this article.

### 3.2 Market for Mortality Risk – An Overview

The first direct mortality risk securitization was a mortality risk bond issued by Swiss Re in December 2003. The bond offered an above-average coupon rate, with the principal payment being dependent on the realized value of a weighted index of mortality rates in five countries (see, e.g. Blake et al., 2006a, and Cowley and Cummins, 2005, for details). The bond (with a maturity of three years) was a short-term
principal-at-risk bond designed to be a hedge for the issuer against catastrophic mortality deterioration over the bond’s life. The bond was issued via a special purpose vehicle named Vita Capital. The face value of the issue was $400m, with the bond fully subscribed.

In April 2005 Swiss Re issued another mortality bond through an SPV, Vita Capital II. The bond had a principal of $362m, was issued in three tranches and was set to mature at 2010. The bond was oversubscribed, indicating again solid market interest.

Lane and Beckwith (2005, 2006) report several other recent mortality-linked issues by Swiss Re (Queensgate, 2005 and ALPS II, 2006) and Scottish Re (Orkney Holdings, 2005). These transactions involved securitization of entire blocks of business, bundling underwriting, business, interest rate and mortality risks. This means that investors in these do not gain pure mortality exposure, which was a major contributor to the success of the Vita Capital issues (Blake et al., 2006a).

In November 2006, AXA issued Osiris, a mortality bond covering extreme mortality in France, Japan and the United States (Lane and Beckwith, 2007). In February 2008 Munich Re established a bond programme, totalling $1.5 billion, for the transfer of catastrophic mortality risk to capital markets. The first series of $100m principal-at-risk bonds with five years maturity was issued through a special purpose vehicle Nathan Ltd. and managed by JPMorgan, covering exceptional mortality events in the United States, Canada, England and Wales, and Germany (see www.artemis.bm).

All of the issues outlined above are mortality bonds, enabling the issuer to hedge short-term, catastrophic mortality risk (similar to CAT-bonds covering various catastrophic events, such as earthquakes and hurricanes). In contrast, there have been practically no transactions directly addressing longevity risk. Longevity bonds were first proposed by Blake and Burrows (2001)⁵, and the first longevity bond was the EIB/PNB Paribas bond announced in November 2004. This bond was to be issued by the European Investment Bank (EIB), with BNP Paribas as the structurer and manager, and Bermuda-based Partner Re as the longevity risk reinsurer (Figure 3). The issue size was £540m, and the

⁵ The authors originally called them survivor bonds, and argued that these should be issued by government to help hedge mortality risk, in the same vein that governments issue inflation-linked bonds. They note that there actually exists a historical predecessor to the proposed structure, namely the 1759 Geneva Tontine.
The bond had an initial coupon of £50m and a maturity of 25 years. The innovative feature was to link the coupon payments to a survivor index based on the realized mortality experience of the population of English and Welsh males aged 65 in 2003. However, the bond was only partially subscribed, and was later withdrawn. The consensus seems to be that this failure to set off was due to problems in bond design, rather than in the type of security itself (see Blake et al., 2006a, for a detailed discussion).

Why have there not been any successful longevity-linked issues to date? An immediate explanation would be to suggest overpricing; however, studies that have considered the first attempt to issue a longevity bond, the EIB/PNB Paribas case discussed above, find that it was more likely underpriced than overpriced (for example, Cairns et al., 2005). Antolin and Blommestein (2007) discuss various possible explanations for the absence of direct longevity securitizations so far, examining the market in the UK. They note that asset-liability matching rules and other regulatory requirements have not been effective in encouraging pension funds to hedge aggregate longevity risk, as there have been no regulatory benefits from using securities for this purpose. Pension funds also generally use current mortality tables without forecasts, only adjusting mortality assumptions every 10 years with new tables. There is therefore a lack of sufficient
incentives to internalize the costs brought about by unexpected gains in longevity.

Antolin and Blommestein (2007), among others, also argue that pension fund managers and trustees did not seem to understand the utility of the longevity-linked product, founding it difficult to comprehend. Pension funds have to investigate all the potential economic, financial, regulatory and other consequences that this kind of new product could have, as well as to assess how the instrument would fit into the overall portfolio. This is a lengthy and potentially costly process, hindering the easy adoption of innovative new products. In general, the pensions industry is usually considered to respond slowly to changes in its environment.

Aside from demand-related factors, a possible reason why longevity bonds or similar instruments have not been successfully offered yet is the lack of standard methodologies for modelling and valuating mortality derivatives. Looking back in history, the credit risk derivatives market, for example, took off after the technology to evaluate credit risk had reached a level adequate to sustain a market (see Dowd et al., 2006).

3.2.1 Outlook and Future Prospects

Recently, there have been some efforts to create mortality or longevity indexes that could serve as transparent and widely accepted benchmarks for mortality-linked transactions. Examples of these are the *LifeMetrics* platform and index introduced by JPMorgan and *QxX Index* by Goldman Sachs. These investment banks have also installed trading desks for longevity risk (Bauer et al., 2008). In March 2007, the Institutional Life Markets Association (ILMA) was founded in New York by Bear Stearns, Credit Suisse, Goldman Sachs, Mizuho International, UBS and WestLB AG. The aim of the association is to “encourage best practices and growth of the mortality and longevity related marketplace”, applying capital market solutions in life insurance, raising awareness, and educating consumers, investors and policymakers about the benefits of mortality and longevity related marketplace (see

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6 “Is the pensions industry burying its head in the sand?” article available at www.pensions-institute.org.
7 Currently the index covers data from the U.S., England & Wales, Germany and the Netherlands. See www.jpmorgan.com/lifemetrics for more information.
8 See http://www.qxx-index.com. This index covers at present only the U.S. senior insured population over the age of 65, but with greater detail and frequent publishing of mortality information.
Loeys et al. (2007) state that, for a new market to succeed, it must provide effective exposure to a risk that is economically important and cannot be hedged through existing markets, and it must have the technology to create liquidity. They conclude that these apply to longevity today. In the same vein, Kabbaj and Coughlan (2007), among others, conclude that a secondary market for longevity risk transfer is emerging, and that the likelihood of success for this market is high. Likely participants in this market include insurance companies, pension funds, investment banks, hedge funds, and asset managers (Figure 4).

Before the Swiss Re mortality bond in 2003, life insurance securitizations were not designed to manage mortality risk, but rather to convert future life insurance profits into cash to increase liquidity. In contrast, new mortality-linked securitizations focus on the liability side of an insurer’s balance sheet, i.e. liabilities on future mortality payments, enabling the insurer to transfer these risks from its liabilities into capital markets. This has the potential to free up capital and increase the competitiveness of an insurance
company. Loeys et al. (2007) argue that the eventual creation of a new market for mortality risk will increase transparency and liquidity, leading to more efficient pricing and distribution of risk, and allowing the cost of longevity hedging to fall from current state. They conclude that the US, UK, and the Netherlands will likely to be the first locations to see the birth of the longevity market, given their market awareness of longevity risk, high-quality mortality indices, prominent pension funds, and regulatory pressure.

4 Longevity Bonds

4.1 Survivor Index

Longevity bonds (LBs for short) are instruments, whose payoffs \( f(t, S(t)) \) are linked to the realized mortality of an underlying reference population, represented by a survivor index, \( S(t) \). Put simply, this index represents the proportion of initial population alive at some future time. We restrict our examination to cases where the reference population is based on a single cohort, e.g. Finnish males aged 65 at the issue of the bond. The survivor index \( S(t,x) \) associated with a cohort initially aged \( x \) is defined by

\[
S(t,x) = \exp\left[-\int_0^t \mu(u,x+u)du\right], \tag{4}
\]

with \( S(0,x) = 1 \). Using assumption (2) and equation (3) derived from this assumption, we get a relation that can be used to easily calculate the survivor index from yearly mortality rates (or survival probabilities):

\[
S(t,x) = \prod_{u=0}^t (1 - q(u,x+u)) = \prod_{u=0}^t p(u,x+u). \tag{5}
\]

The survivor index represents the proportion of the reference population cohort (aged \( x \) at time 0) that are expected to be alive at some future time \( t \): if \( \mu(t,x) \) is deterministic, \( S(t,x) \) equals the probability that an individual aged \( x \) at time 0 will survive to age \( x + t \). If \( \mu(t,x) \) is stochastic (as assumed in this article) then, looking from time 0, \( S(t,x) \) becomes a random variable observable only at time \( t \). In this case, we have to work with
the expectation of \( S(t,x) \). To formalize the foregoing discussion, let \( I(t) = 1_{\tau = t} \) be the indicator function that changes from 1 to 0 at the time of death \( \tau \) of an individual. Let \( \mathcal{F}_t, t \geq 0 \), be the filtration generated by the force of mortality process up to time \( t \), that is, \( \mathcal{F}_t \) represents all the information about the history of the process available at that time. Then \( S(t,x) \) is \( \mathcal{F}_t \)-measurable, and

\[
\Pr[I(t) = 1|\mathcal{F}_u] = E[S(t,x)|\mathcal{F}_u] = S(t,x) \quad \forall u \geq t.
\]

For \( u \in (0,t) \), \( S(t,x) \) is a random quantity as pointed out earlier, and we have

\[
\Pr[I(t) = 1|\mathcal{F}_u] = E[S(t,x)|\mathcal{F}_u] = S(u,x)E\left[\frac{S(t,x)}{S(u,x)}|\mathcal{F}_u\right].
\]

Following Cairns et al. (2006a, 2008a) we define the spot survival probabilities

\[
p(u,t,x) = \Pr[I(t) = 1|I(u) = 1, \mathcal{F}_u] = E\left[\frac{S(u,x)}{S(t,x)}|\mathcal{F}_u\right]. \tag{6}
\]

using which the previous relation can be expressed as \( \Pr[I(t) = 1|\mathcal{F}_u] = S(u,x) p(u,t,x) \): that is, the probability, as seen from time \( u \), that an individual aged \( x \) at 0 will survive to time \( t \) is equal to the probability that the individual has survived until \( u \) multiplied by the spot survival probability at time \( u \) of surviving to a future time \( t \). In addition to spot survival probabilities, we define forward survival probabilities as follows:

\[
p(t,T_0,T_1,x) = \Pr[I(T_1) = 1|I(T_0) = 1, \mathcal{F}_t]. \tag{7}
\]

Forward survival probability (7) is the probability, as measured at time \( t \), that an individual aged \( x \) at time 0 and still alive at \( T_0 \) will survive until time \( T_1 > T_0 \). We have to resort to this rather lengthy notation for the arguments of \( p(\cdot) \) to express the foregoing notions clearly\(^9\). It may be useful at this point also to note that the spot and forward survival probabilities are related to individuals aged \( x \) at time 0, not at time \( t \). In contrast, the simple survival probabilities \( p(t,x) \) introduced earlier concern individuals

\(^9\) Let us mention that in commonly used actuarial notation (see e.g. Bowers et al., 1997), the probability of a live aged \( x \) at time 0 surviving to age \( x + t \) is denoted \( p_x^t \); that is, \( p_x^t = p(t,0,x) \).
aged $x$ at time $t$: $p(t, x) = p(t + 1, x - t)$.

### 4.2 Types of Longevity Bonds

Longevity bonds can take a variety of forms, depending on the type of bond, survivor index chosen, specification of the payment function $f(t, S(t, x))$, maturity, credit risks involved, position to be hedged, and institution and portfolio type (e.g., life insurance or annuity contracts). We examine some of these possibilities here. The following is based on Blake et al. (2006b):

- **Standard LBs**: these are coupon-bearing bonds, whose coupon payments are linked to a survivor index and fall over time in line with the index. A simple example of a payment function could be $f(t, S(t, x)) = kS(t, x)$, for some $k > 0$.
- **Inverse LBs**: unlike standard LBs, the coupon payments of these are inversely related to a survivor index, rising over time instead of falling. In terms of the previous example, a payment function for inverse an LB might be of the form $f(t, S(t, x)) = k(1 - S(t, x))$, $k > 0$.
- **Longevity Zeros**: these are zero coupon bonds, whose principal payments are functions of a survivor index. Longevity zeros could serve as building blocks for more complicated securities.
- **Principal-at-risk LBs**: these are longevity bonds, whose coupon payments (fixed or floating) are not affected by mortality development, but whose principal payment is linked to a survivor index.
- **Survivor bonds**: unlike standard longevity bonds, these have no specified maturity but continue to pay coupons as long as any individual belonging to the reference population is still alive: that is, the bonds have a stochastic maturity equal to the time of death of the last member of the reference population. There is no principal payment.
- **Collateralized longevity obligations (CLOs)**: similar to conventional collateralized debt obligations (CDOs), CLOs would be tranches on pools of longevity bonds. Different tranches would have different exposures to longevity risk, and therefore different expected returns associated with them. See Wills and Sherris (2008) for an analysis of pricing tranched longevity risk.
4.2.1 Financial Engineering

Blake et al. (2006b) discuss four different ways to construct longevity bonds. The first is to decompose the cash flows of a conventional bond into two longevity-dependent instruments, a longevity bond that pays coupon equal to $S(t,x)$, and an inverse longevity bond that pays coupon equal to $1 - S(t,x)$.

Another approach is to combine longevity zeros and a longevity swap. This involves using a series of $T$ zero coupon bonds of increasing maturity combined with a $T$-year longevity swap (with $T$ being the desired maturity of the LB). The floating leg of the swap involves a payment $S(t,x)$ dependent on the realized survivor index, and this is the amount paid to the holders of the longevity bond. The fixed leg is set so that the value of the swap is zero initially. The fixed leg of the longevity swap and the payments from longevity zeros can be made to cancel each other, producing the same payoff, $S(t,x)$, as a longevity bond.

A third way to engineer a longevity bond is similar to the previous one, but this time a series of $T$ longevity zeros is combined with a series of forward contracts. The forward contracts are used to exchange the survivor index payoff $S(t,x)$ for its forward price in each year $t$. The one-year forward price at $t - 1$ is the expected (spot) price of $S(t,x)$ at $t$, and the one-year longevity zero maturing at $t$ is chosen so that its payoff offsets the forward price, again producing a net payoff of $S(t,x)$.

A fourth way to construct an LB is to combine a conventional long-term coupon-bearing bond with a put option on the principal, with the option’s expiry date set equal to the bond’s maturity date. The option is used to hedge the tail risk, i.e. the risk that a large proportion of the underlying reference population lives longer than anticipated. One major difficulty with this approach is the lack of very long-term bonds. For a more detailed discussion regarding financial engineering of longevity bonds, we refer the reader to the previous reference.

4.3 Pricing of Longevity Bonds

We adopt risk-neutral approach to the pricing of longevity bonds (see, e.g. Dahl, 2004,
and Dahl and Møller, 2006). Up to this time, we have implicitly used the real-world or true probability measure, denoted by \( P \). To price LBs, we use a risk-neutral probability measure \( Q \) (also referred to as the equivalent martingale measure) that is in a probabilistic sense equivalent to the real-world measure \( P \). A change of probability measure is a commonly employed method in the pricing and valuation of contingent claims. Within no-arbitrage pricing theory framework, the price of a contingent claim is obtained as the expected payoff with respect to a risk-neutral probability measure (Harrison and Kreps, 1979).

Pricing a derivative security in complete markets involves replicating portfolio. As Cairns et al. (2006b) note, at present the market for mortality-linked instruments is not in a state where all contingent claims could be replicated using dynamic hedging strategies. Due to the incompleteness of markets\(^{10}\), the risk-adjusted measure \( Q \) is not unique. We will consider the dynamics under \( Q \) in Chapter 5 when specifying the model used for forecasting mortality.

As stated, the basic idea behind risk-neutral valuation is that a derivative dependent on some variable(s) can be valued by calculating the expected payoff with respect to the risk-neutral world and discounting the expectation at the risk-free rate. There are many possible risk-neutral worlds that can be assumed in a given situation, depending on the assumption about the market price of risk. The market price of risk associated with a given variable determines the growth rates of securities that are derivatives of that variable (see, e.g. Hull, 2003 [Chapter 21]). For some choice of market price of risk, we obtain the real-world probability measure, and the growth rates coincide with those observed in reality.

The extent to which \( P \) and \( Q \) differ depends, therefore, on the value of risk price parameter. This is reflected in, for example, the amount of premium that insurers and other institutions are willing to pay to hedge their (systematic and possibly unsystematic) mortality risks (Cairns et al., 2008a).

We make the conventional assumption that unsystematic mortality risk does not attract

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\(^{10}\) In an uncertain world, an economic agent could eliminate financial risk it is facing, if it were possible to obtain a perfect hedge against any possible event or state of nature. Such a market is defined as complete, as contingent claims can be written on every state of world without restriction (Schlesinger and Doherty, 1985).
a risk premium, i.e., the market price of risk for unsystematic risk is zero. We also assume that the dynamics of mortality and the term structure of interest rates are independent. The price at time 0 of a longevity bond that pays $S(t,x)$ at time $t$, $t = 1, \ldots, T$, is

$$V(0) = E_Q \left[ \sum_{t=1}^T \exp\left(-\int_0^t r(u)du\right)S(t,x) \bigg| \mathcal{F}_0 \right],$$

where $\mathcal{F}_0$ represents the information about mortality process available at time 0, $r(t)$ is the (risk-free) interest rate process, and $E_Q \left[ \cdot \bigg| \mathcal{F}_0 \right]$ is the expectation with respect to the risk-neutral measure $Q$, conditional on $\mathcal{F}_0$. Assuming that interest rates and mortality are independent, (6) can be written

$$V(0) = \sum_{t=1}^T P(0,t)E_Q \left[ S(t,x) \bigg| \mathcal{F}_0 \right],$$

where $P(s,t)$ is the price of a zero-coupon bond issued at time $s$ and paying 1 at time $t \geq s$,

$$P(s,t) = E_Q \left[ \exp\left(-\int_s^t r(u)du\right) \bigg| \mathcal{F}_s \right].$$

Using (9), we can price a longevity bond by taking the expectation of $S(t,x)$ with respect to a risk-adjusted probability measure $Q$, discounting with the price of an ordinary zero-coupon bond maturing at time $t$, and summing over all $t$.

In practice, there are significant difficulties involved in the pricing of longevity bonds. Presently the market is not liquid and therefore the longevity risk is not easily tradable. The implication of this is that the calibration of the model in the usual financial market sense using market data is not possible. Another major issue is the need to accurately forecast future mortality in order to price the bond.

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11 In discrete time, this can be written as $P(s,t) = E_Q \left[ (1 + r(t))^{t-s} \bigg| \mathcal{F}_s \right]$. 
4.4 A Simple Hedging Framework

Let us examine a case where an insurer wants to hedge its exposure to longevity risk present in its annuity book. Let $A(t_0, x)$ be the total number of annuities issued to the initial cohort consisting of individuals aged $x$ at time $t_0$, and assume that the insurance contract involves a single premium payable at the beginning. This premium is based on the insurer’s expectation regarding the annuity payments it will have to pay, $\overline{CF}(t)$, as seen from time $t_0$:

$$\overline{CF}(t) = A(t_0, x)p_{t,t_0,t,x}^{\text{ref}}, \quad (10)$$

where $p_{t,t_0,t,x}^{\text{ref}}$ is the survival probability based on the reference lifetable used by the insurer for pricing and reserving. The actual payout of the insurer at time $t$ (assuming payments of 1 unit) is

$$CF(t) = A(t_0, x)p(t,t_0,t,x). \quad (11)$$

where $p(t,t_0,t,x)$ is the realized survival probability (i.e., the probability as measured at time $t$ that an individual aged $x$ and alive at time $t_0$ survives until time $t$). If the number of insured who live to old ages is greater than that assumed when pricing contracts, the insurance company ends up making losses. The loss on the insurer’s portfolio at time $t$ can therefore be written

$$L(t) = CF(t) - \overline{CF}(t) = A(t_0, x)(p_{t,x} - p_{t,x}^{\text{ref}}), \quad (12)$$

where we have adopted the shorter notation for survival probabilities for the sake of convenience. If the reference probabilities provide an unbiased estimate of the true survival probabilities, then

$$p_{t,x}^{\text{ref}} = E[\ast, p_{t,x}]. \quad (13)$$

Suppose that, at time $t_0$, the insurer buys a longevity bond\(^\text{12}\) linked to an appropriate survivor index and finances this by issuing a fixed-rate bond of equal maturity (say, $T$).

\(^{12}\) Alternatively, it could issue an inverse longevity bond.
Let $N(t_0, x)$ be the nominal value of the bonds, and denote the coupons paid by the longevity bond and by the fixed bond by $C(t)$ and $K$, respectively. At time $t$, $0 < t \leq T$, the difference between the coupons received and the coupons paid (floating and fixed leg, respectively) is

$$\Delta(t) = N(t_0, x) \left( C(t) - K \right). \tag{14}$$

The longevity risk would be perfectly hedged if $L(t) = \Delta(t)$, that is, possible losses on the insurance portfolio are exactly offset by the profit arising from the bond position (and vice versa). The details concerning longevity bonds and standard bonds needed to set up a hedge depend critically on the survival probability forecast used by the insurer and the uncertainty related to it. If (13) holds, then from (12) the expected loss is zero by construction, as the insurer can predict the future mortality rates without bias (i.e., the expectation hold is an unbiased estimate of the true mortality). However, the randomness in the portfolio loss due to the random survival probability still remains. The insurer’s aggregate cashflow, $y(t)$, at time $t$ can be written

$$y(t) = \Delta(t) - L(t) = A(t_0, x)(., p_x - E[., p_x]) - N(t_0, x) \left( C(t) - K \right). \tag{15}$$

Assuming that the coupon payments of the LB are directly proportional to the survivor index\(^{13}\), that is, $C(t) = kS(t, x)$ for a pre-specified amount $k$, the variance of the cashflow is

$$\text{Var}[y(t)] = A^2 \text{Var}[., p_x] - N^2k^2 \text{Var}[S(t, x)] - 2ANk \text{Cov}[., p_x, S(t, x)]. \tag{16}$$

where the arguments from $A$ and $N$ have been dropped for brevity. We can now construct a minimum-variance hedge by minimizing the variance expressed in (16). By setting the derivative with respect to $N$ zero, we get

$$N = \frac{ACov[., p_x, S(t, x)]}{\text{Var}[S(t, x)]}. \tag{17}$$

If there is no basis risk, then at time $0$ $S(t, x) = . p_x$. In this case, (17) reduces to

\(^{13}\) This was the case with the EIB/PNB Paribas longevity bond, whose coupon payments were $C(t) = \£50m \cdot S(t)$. 

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that is, with perfect correlation between the index and the insurer’s mortality experience, the hedge reduces to setting up the bond transactions so that the total nominal amount is equal to the product of the fixed amount and the annuity liabilities.

The standard procedure for estimating minimum variance hedge ratios is to regress changes in the returns of the asset or portfolio to be hedged on historical changes in the price of the hedging instrument, over time horizon matching the length of the considered hedge (Ederington and Salas, 2006). This approach applies to the foregoing analysis as well – where we consider an insurance portfolio with an underlying mortality rate and exposure that is being hedged using a longevity bond – by replacing returns with survival probabilities.

4.5 Complications in Using LBs as Hedging Instruments

Aside from the problems of market incompleteness and the difficulty of forecasting mortality, there are several other issues that have to be considered when evaluating or designing longevity bonds.

The need to forecast uncertain future mortality involves parameter and model risk. Parametric risk arises because of limited sample sizes available for use in the estimation of model parameters. See Cairns et al. (2006b) for an examination of parameter uncertainty in a mortality modelling framework. Model risk pertains to the chosen model, as several models may fit the limited historical data adequately, and the identification of a single “correct” model may be difficult or even impossible. Cairns et al. (2008a) discuss various model selection criteria that can be used to evaluate given mortality model.

Aside from model-related factors, there are several possible problems regarding mortality data itself, such as accuracy, reliability, availability and lack of timeliness. Mortality data is usually published, at best, annually and in some countries much less frequently. The lags in the production, adoption and disclosure of mortality tables have to be taken into account when designing particular mortality-linked securities. In addition, the mortality index chosen should be as transparent as possible, providing investors a ready access to all relevant information. The underlying index must also be
perceived to have integrity in the way it is computed. As Blake et al. (2006a) note, the less information investors have, the more risky the security to in question may appear, turning away potential investors.

Moral hazard can also arise in the context of mortality-linked securities, when one party in a transaction (for example, data providers) has significantly earlier access to the data than investors have. Another possible situation involving moral hazard concerns the chance of the data underlying the reference index being manipulated, as the possible application of smoothing techniques for crude mortality data makes the process less straightforward and transparent (Blake et al., 2006a).

An important issue related to the construction of a survivor index is the choice of a reference population. To provide an effective hedge, the reference index to which the bond’s payments are linked should match the mortality experience of the hedger’s portfolio closely enough. That is, the basis risk arising from differences in mortality between the index and the insured population should be minimized. As pointed out by, for example, Blake et al. (2006b), this implies that a large number of longevity bonds might need to be issued, with the underlying survivor indexes covering the full range of ages and both genders. This, on the other hand, would make it hard for a liquid market to develop. A viable market in longevity instruments needs to attract a sufficient amount of speculators to bet on positions opposite to hedgers and to ensure that arbitrage opportunities are eliminated. The choice of reference index thus needs to weight liquidity considerations against the emergence of basis risk.

The basis risk (arising from the reference population mismatch and the maturity mismatch) between a survivor index and insurer’s own longevity exposure is a problem, but this problem is, more or less, present in all index-linked transactions. However, there are standard ways of dealing with the basis risk: once the insurer compiles its own mortality history for different cohorts, it can obtain the historical correlation between its own position and the index to be used in hedging calculations (see equation (17)). It can then use a mix of contracts that best matches its exposure, in the same way as when hedging conventional interest rate risk.
5 Modelling Stochastic Mortality

To make the pricing of longevity bonds operational, models for the development of mortality and interest rates need to be specified. In this article, we assume that the realized mortality rate is governed by the following model by Cairns et al. (2006b, 2007)

\[
\log \frac{\tilde{q}(t, x)}{1 - \tilde{q}(t, x)} = \kappa_1(t) + \kappa_2(t)(x - \bar{x})
\]

or, equivalently,

\[
\tilde{q}(t, x) = \frac{\exp\left[\kappa_1(t) + \kappa_2(t)(x - \bar{x})\right]}{1 - \exp\left[\kappa_1(t) + \kappa_2(t)(x - \bar{x})\right]},
\]

where \( \tilde{q}(t, x) = 1 - \tilde{p}(t, x) = 1 - p(t+1, t+1, x) \) is the realized mortality rate for the cohort aged \( x \) at time 0, defined through the corresponding realized survival probability \( \tilde{p}(t, x) \). \( \kappa_1(t) \) and \( \kappa_2(t) \) are stochastic processes assumed to be measurable at time \( t \), and \( \bar{x} \) is the mean age over the age range used in the specification of the model. The first stochastic factor, \( \kappa_1(t) \), affects mortality at all ages in an equal way, whereas the effect of the second factor, \( \kappa_2(t) \), is proportional to age.

The chosen model (thereafter referred to as CBD), based on the logistic transform of the mortality rate, is a special case of so called Perks stochastic models. Cairns et al. (2006b) show that this model provides a good fit to mortality data for English and Welsh males covering years 1961 to 2002. For a comparison of various stochastic mortality models proposed in the literature, see Cairns et al. (2007, 2008b, 2008a). Despite its relative simplicity, the CBD-model performed well in the forecasting experiments conducted in the first two of the above-mentioned papers; its results were also concluded to appear robust and biologically reasonable\(^{14}\).

\(^{14}\) An example of biological reasonability is the requirement the mortality rates for the elderly do not fall with age. Biological reasonability is, to some extent, a judgmental question. Cairns et al. (2007) point out that experts in the field of mortality hold certain subjective views about how mortality might evolve in the future, or how mortality rates at different ages should relate to each other, and not all of them agree with each other. Pesonen (2004) points out that mortality operates within a complex framework and is influenced by a multitude of biological variables, socioeconomic and environmental factors, government policies, health conditions and behaviors, and other factors.

See Cairns et al. (2008a) for discussion concerning model selection criteria, including biological
To forecast the development of mortality, we assume that $\mathbf{k}(t) = \left( \kappa_1(t), \kappa_2(t) \right)^T$ is a 2-dimensional random walk with drift:

$$\mathbf{k}(t) = \mathbf{k}(t-1) + \mathbf{\mu} + \mathbf{C} \mathbf{Z}(t), \quad (20)$$

where $\mathbf{\mu} \in \mathbb{R}^{2d}$ is a constant drift vector, $\mathbf{C} \in \mathbb{R}^{2 \times 2}$ is a constant lower triangular matrix and $\mathbf{Z}(t) \in \mathbb{R}^{2d}$ is a vector of independent standard normal random variables, $Z_1(t), Z_2(t) \sim N(0,1)$.

Taking $t_0$ as the starting point, we can write the process (20) at time $t_0 + k$ as

$$\mathbf{k}(t_0 + k) = \mathbf{k}(t_0) + k\mathbf{\mu} + \sum_{j=1}^{k} \mathbf{C} \mathbf{Z}(t + j). \quad (21)$$

Taking expectation of this, we get the point estimate at $t_0$ of the random walk process at future time $t_0 + k$, i.e.,

$$E \left[ \mathbf{k}(t_0 + k) \mid \mathcal{G}_t \right] = \mathbf{k}(t_0) + k\mathbf{\mu}, \quad (22)$$

where $\mathcal{G}_t$ represents all the information about the process $\mathbf{k}(t)$ up to and including time $t$.

Equation (20) describes the dynamics of the random-walk process $\mathbf{k}(t)$ under the real-world probability measure $P$. Under the risk-adjusted probability measure $Q = Q(\lambda)$, we write (c.f. Cairns et al., 2006b)

$$\mathbf{k}(t) = \mathbf{k}(t-1) + \mathbf{\mu} + \mathbf{C} \left( \mathbf{\bar{Z}}(t) - \lambda \right) = \mathbf{k}(t-1) + \tilde{\mathbf{\mu}} + \mathbf{C} \mathbf{\bar{Z}}(t), \quad (23)$$

where $\tilde{\mathbf{\mu}} = \mathbf{\mu} - \mathbf{C} \lambda$, $\mathbf{\bar{Z}}(t) = (\bar{Z}_1(t), \bar{Z}_2(t))^T$ is a two-dimensional standard normal random variable under $Q$, and $\lambda = (\lambda_1, \lambda_2)$ represents the market price of longevity risk. Specifically, $\lambda_1$ is the market price of risk associated with the process $Z_1(t)$ and $\lambda_2$ the market price of risk associated with the process $Z_2(t)$. Cairns et al. (2006) note that

plausibility.
with the specification chosen, \( \lambda_1 \) is related to level shifts in mortality (or, to be accurate, in the curve \( \log \left( \frac{1}{q} \right) \)) and \( \lambda_2 \) is related to a tilt in the curve.

We assume that the market price of risk \( \lambda \) is constant: as Cairns et al. (2006b), for example, point out, it is difficult to propose any form more sophisticated for \( \lambda \) given the absence of market price data. Bauer (2006) speaks of a “market price of mortality puzzle”: most authors and practitioners seem to agree that there is, or should be, a risk premium for systematic mortality risk, but there is no accordance of its form or size.

6 Numerical Illustration

6.1 Description of the Data

We use data for Finnish males and females, separately, obtained from the Human Mortality Database\(^{15}\) (HMD). The data contains numbers of deaths \( D(t,x) \) and corresponding exposures (i.e., population sizes) \( E(t,x) \) by gender, age, and year of birth. These are used to calculate crude death rates. Figures 5 and 6 show the crude death death rates \( m(t,x) \) and the expected death rates \( \hat{m}(t,x) \) (obtained from the model) for males and females, respectively. We can see from the figures that mortality in general has been improving over the observation period for both males and females, but in a seemingly stochastic fashion. At higher ages the volatility of mortality also increases notably, but this volatility, too, has been decreasing towards the end of the observed years.

We exclude years prior to 1955 from our examination, following Mäkinen (2004), who notes that Finnish mortality rates during and after war years deviate significantly from the general trend, causing bias in parameter estimation. As our purpose is to forecast mortality development from current time onwards, we use data covering years from 1955 to 2006 (latest year available at the time of writing) and ages from 60 to 95 in estimating the parameters for our model. Age cohorts with fewer than five observations

\(^{15}\) Data available at www.mortality.org. The data used in this article was downloaded in July 2008.
are excluded from the data.

Figure 5. Crude death rates (left) and fitted death rates (right) for Finnish males.

Figure 6. Crude death rates (left) and fitted death rates (right) for Finnish females.

6.2 Model Calibration

For each calendar year $t = 1955, \ldots, 2006$, we will estimate the parameters of the model using maximum likelihood. The number of deaths is modelled using the Poisson distribution commonly employed in mortality modelling literature (see, e.g. Brouhns et
al., 2002). We thus assume that $D(t,x) \sim \text{Poisson}(E(t,x)m(t,x))$. In addition, we employ the assumption that $m(t,x) = \mu(t,x)$ often used in the analysis of death rate data (see Cairns et al., 2007 and the previous reference). Using this and equation (3), we can write

$$m(t,x; \kappa_1, \kappa_2) = -\log \left[1 - q(t,x; \kappa_1, \kappa_2)\right],$$

where the dependence on the model parameters is explicitly expressed. Now the log-likelihood function becomes (Cairns et al., 2007)

$$l(\kappa_1, \kappa_2; D,E) = \sum_{t,x} D(t,x) \log \left[E(t,x)m(t,x; \kappa_1, \kappa_2)\right] - E(t,x)m(t,x; \kappa_1, \kappa_2) - \log \left[D(t,x)\right]$$

(24)

The parameters $\kappa_1(t)$, $\kappa_2(t)$ are estimated by maximizing (24) numerically. Figures 7 and 8 show the results for males, and Figures 9 and 10 for females.
Figure 8. Estimated values for parameter $\kappa_2$ for males.

Figure 9. Estimated values for parameter $\kappa_1$ for females.
The figures show clear trends in both parameters, and these are very similar for both genders. Cairns et al. (2006b) get similar results using data for English and Welsh males above age 60 covering years 1961–2002. As they note, the decreasing trend in \( \kappa_1(t) \) indicates general improvements in mortality at all ages. The increasing trend in \( \kappa_2(t) \) shows that the slope of the curve is getting steeper, or that mortality improvements have been greater at lower ages. The results also show that the trends in \( \kappa_1(t) \) and \( \kappa_2(t) \) are steepening after years 1970 and 1980, respectively. This is evident for both males and females.

Figures 11 and 12 show the crude death rates versus fitted death rates for the most recent year of observation (2006), for both genders. These figures show that the fit provided by the model is quite good.
Figure 11. Crude death rates (circles) and fitted death rates (solid line) in year 2006 for males.

Figure 12. Crude death rates (circles) and fitted death rates (solid line) in year 2006 for females.
6.3 Simulation Results

We consider a 30-year longevity bond whose coupon payments are linked to a survivor index \( S(t) \) for a cohort of Finnish individuals aged \( x = 63 \) in year 2006, treating males and females separately. Monte Carlo simulation with \( N = 10000 \) trials is used to obtain the distribution of \( \hat{q}(t, x) \) for each \( t = 1, \ldots, 30 \) by simulating the underlying random-walk process \( \kappa(t) \), and values for \( S(t) \) are calculated from these using (5).

The value of the longevity bond is obtained from equation (9). As our focus in this paper is in mortality-related risk, we make the simplifying assumption that the term structure of interest rates is flat at 4.00%. With this assumption, we get \( V(0) = 12.268 \) for males and \( V(0) = 14.511 \) for females as the price\(^{16} \) of the longevity bond using data from calendar years 1952–2006. If, instead, we use data pertaining to the last 26 years only (because of the steepening trend after 1980, discussed in relation to model calibration), we get a price of 12.941 for males and 14.519 for females. This reflects the generally improving mortality over the observation period: as greater proportions of individuals survive to old ages, the coupon payments increase in line, making the bond more valuable. It is interesting to note that, although the survival properties for women are higher than those for men, the probabilities have increased much more for men than for women when estimated using the last 26 years versus 52 years of data. Figures 13 and 14 show the survivor indexes and associated 95% confidence intervals for males and females, respectively.

In complete markets, we would calculate security prices choosing such model parameters that the model-endogenous prices match the prices quoted in the market optimally. But because there is no liquid market for longevity-linked securities, it is not possible to rely on market data for pricing purposes and comparison in practice. Without a benchmark, it is difficult to draw conclusions from the obtained prices or to compute market prices of risk.

\(^{16}\) Ignoring the deterministic present value of the principal.
Figure 13. Expected values (solid line) and 95% confidence intervals (dotted lines) of $S(t)$ for males, using 52 years of data (left panel) and 26 years of data (right panel) in simulation.

Figure 14. Expected values (solid line) and 95% confidence intervals (dotted lines) of $S(t)$ for females, using 52 years of data (left panel) and 26 years of data (right panel) in simulation.
To make some inferences regarding the bond prices, we use the reference mortality K2004 for life insurers accepted by the Finnish Actuarial Society in 2005 as our reference point. We take this to be the lifetable used by companies in pricing annuities. With these mortality projections, we obtain prices of 12.258 and 14.121 for males and females, respectively. If we take that the CBD-model provides an unbiased estimate of the rate of mortality improvement among the population, then insurers using the K2004 projections to price longevity bonds (or annuities without using any conservative margin) appear to underestimate the improvement in mortality and thus the value of longevity-linked instruments or annuities. For males, this underpricing is quite small, the percentages being 0.23% using 52 years of data and 1.87% using the last 26 years of data. For females, the underpricing is somewhat larger: 2.69% using 52 years of data and 2.74% using the last 26 years of data.

However, we point out that one should exercise appropriate caution before drawing strong conclusions from the results of the foregoing analysis. Factors such as possible differences between the mortality of the insured population compared to the population as a whole may affect the results. More importantly, we are in essence comparing the projections obtained from two models, the current deterministic reference mortality used within insurance industry and the stochastic CBD-model used in the simulations of this article. Therefore, the validity of the results depend on the validity of the chosen model. In any case, it is apparent that the K2004 projections seem to underestimate the rate of mortality improvements, and therefore the longevity risk, at higher ages when compared to the stochastic forecasts. The use of stochastic analysis and simulation also makes it straightforward to gauge and explicitly quantify the amount of uncertainty related to a forecast.

7 Conclusions

In addition to interest rates and inflation, unexpected improvements in the lifetimes of the insured population are major source of risk for life insurers and pension plans. In order to address the problems that this longevity exposure creates, we need to quantify

17 Available at www.actuary.fi.
and price the risk. This requires suitable stochastic mortality models to forecast the distribution of future mortality rates. In addition, we need instruments to manage or transfer longevity risk. In this paper, we considered longevity-linked securitizations as a means of managing and hedging longevity risk, concentrating on longevity bonds.

After presenting some concepts related to mortality modelling and mortality securitizations, we provided a brief overview of the publicly announced mortality-linked transactions conducted so far. We discussed the characteristics, use as hedging instruments, and pricing of longevity bonds, and specified the CBD-model used here to forecast future mortality. This model was fitted to Finnish mortality data, covering years 1955–2006 and individuals aged 60–95, males and females separately. The model was shown to provide a good fit to historical mortality data. Assuming the underlying process to be governed by random walk with a drift, we simulated the development of mortality rates to obtain longevity bond prices. The results indicated that the use of the reference mortality projections for life insurers accepted by Finnish Actuarial Society potentially leads to underestimation of the value of longevity bonds (or annuities having similar payment schedule). The survival probabilities obtained from the deterministic projections underestimated those produced by the stochastic model at high ages. This would seem to suggest the importance of using a carefully specified stochastic mortality model in order to provide reliable measures of mortality, and of its uncertainty, for pricing and reserving purposes.
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