AN OLIGOPOLISTIC CONJECTURAL VARIATION MODEL OF THE NORDIC ELECTRICITY MARKET

MAT-2.108 INDEPENDENT RESEARCH PROJECTS IN APPLIED MATHEMATICS
TUUKKA SARVI, 57679S
TKK DEPARTMENT OF ENGINEERING PHYSICS AND MATHEMATICS
DECEMBER 20, 2007
1 Introduction

There have been numerous studies of electricity markets since the deregulation of electricity markets begun 1990s in United Kingdom, Nordic countries and California. Models have been build for example to analyse the extent of market power exercised by large firms in the deregulated electricity markets, to study the spot market price determination in the short run and to study the effect of various policies to consumption and price level in electricity markets. The models used to capture the strategic behavior of firms in electricity markets fall roughly in to the following categories: Cournot-Nash models, supply function equilibrium models, auction models, conjectural variation models and conjectured supply function models.

A popular approach for modelling the electricity markets are Cournot type games of competition in quantities (Cournot-Nash models). One of the first Cournot type models of electricity markets was devised by Bergman and Andersson (1995) [5]. They used a numerical oligopoly model to perform an ex-ante analysis of the relation between the Cournot-equilibrium price in the deregulated Swedish electricity market and the number of firms and size distribution of firms on the market. This approach was continued by Amundsen et al. (1998) who used a numerical multicountry electricity market model to analyse the impact of deregulation and free trade on electricity prices in the Nordic countries [1]. They calculated Cournot and perfect competition equilibria with and without free inter-country trade in electricity. Borenstein et al. (1999) discussed the weaknesses of concentration measures as a viable measure of market power in the electricity industry proposed and alternative method for the analysis of market power based on market simulations with a Cournot type oligopoly equilibrium model and the use of plant level data [7]. More recently, Amundsen et al. (2002) used a numerical Cournot oligopoly model to analyse the effect of cross-ownership on the degree of horizontal market power in the Norwegian-Swedish electricity market [2]. In general, Cournot type models are a conservative way to model electricity markets because the degree of competition in Cournot competition is low compared to other approaches. Much of its popularity can be attributed to the simplicity of the approach and the ease of computation. A drawback to the Cournot type models is that they usually do not give meaningful answers when price elasticity of demand is low (which is often the case in electricity markets in the short run). Furthermore, Cournot type models usually predict that mergers will be unprofitable for the merged firms [10].

Another popular approach for modelling the electricity industry is to calculate so called supply-function equilibria suggested by Klemperer and Meyer (1989) [20]. This approach to model electricity markets has been used by for instance by Bolle (1992) and Green and Newbery (1992) [6, 17]. In this
approach, the strategic variable is a supply function which corresponds to actual POOLCO-type auctions used in some electricity markets. Common to supply-function equilibrium models is an assumption of well-defined institutional framework for bidding and market clearing and that the analysis is focused on the spot market price determination in the very short run. A drawback of the supply-function equilibrium models is that equilibria are difficult to calculate because the optimization problem faced by each firm is nonconvex and can possess multiple local optima [10]. Thus, it can be problematic to model larger networks with supply-function equilibrium models.

One approach is to model electricity markets is to use auction theory suggested by von der Fehr and Hardbord (1993) where electricity spot markets are modeled as sealed bid private value multiple unit single price auctions [27]. This approach has been used for example by Rudkevich et al. (1998) who analysed bidding games in symmetric duopoly and apply the analytical results to a simulation of regional bid based electricity markets in the United States [24]. Auction models bear close resemblance to supply-function equilibrium models. Auction models are an attractive way to model the electricity markets because of the bidding model is very close to the actual market clearing mechanism in many electricity markets. Furthermore, auction models do not share many of the inherent difficulties present in supply-function equilibrium models [30].

Conjectural variation models are a generalized form of Cournot type models. In contrast to Cournot models, in conjectural variation models firms do not assume that their rivals' outputs are fixed. On the other hand, it is assumed firms will anticipate that their rivals will form best response functions and incorporate this into their profit maximization. Conjectural variations models have been theoretically studied for example by Bresnahan (1983) and Lindh (1992) [9, 21]. Their application for modeling electricity markets seems to be uncommon. A problem with the conjectural variations models is the multitude of possible Nash equilibria. However, it has been shown that the multitude of equilibria is not logically inconsistent but rather a product of bounded rationality of agents acting in a dynamic framework [18, 16].

Conjectural supply function models are a mixture of supply-function equilibrium models and conjectural variation models. In this approach it is assumed that firms conjecture the supply functions of their rivals. Conjectural supply function models have been used to model the electricity industry by e.g. Day et al. (2002). They compare the conjectured supply function model to alternative models (e.g. Cournot oligopoly models with quantity setting) and conclude that CSF models are more realistic and flexible for modelling imperfect competition [10].
There have been some comparative studies between different electricity market models. For example, Barquin et al. (2004) investigated the robustness of numerical electricity market models. They compared the results of numerical models of three different research groups for the same data and found out that predicted prices differed significantly. They concluded that simulation results are highly sensitive to various assumptions of the model [3].

In addition to research published in journals, larger scale models for simulation and analysis of the electricity markets have been developed. An example of such is Balmorel (Danish Energy Research Program) [31].

This study presents an oligopolistic conjectural variation model of electricity markets. The model assumes identical generating firms in each region and formulates number of firms in a region as an endogenous variable. The model is based on model by Sulamaa [25] and earlier models by Bergman and Andersson [5]. The purpose of this study is to present the model and investigate some simple scenarios in the Nordic electricity market using data from the year 2006.

2 Nordic electricity market

The Nordic electricity market consists of the national electricity markets of Norway, Sweden, Finland and Denmark. Electricity and related derivatives are traded in the Nordic electricity exchange Nord Pool. Nord Pool consists of daily spot market for physical contracts, financial derivatives market and clearing services for contracts traded in OTC bilateral contracts [28].

The deregulation of Nordic electricity markets began in 1991 with Norway opening its previously strictly regulated market for competition. The process continued in 1996 when a combined Norwegian-Swedish spot market for electricity, Nord Pool, was opened. In 1998 Finland joined Nord Pool completing the deregulation of its electricity market. At the end of the year 2000 Denmark joined Nord Pool [1, 33].

Ten years after its initiation in 2006 the total consumption of electricity in the Nordic electricity market was 395.4 TWh. Total generation of electricity was 383.9 TWh. The difference between consumption and generation was imported from outside the Nordic electricity market [34].

The main types of power generating plants in the Nordic market are: hydro power, nuclear power, combined heat and power (CHP), condensing power, wind power and gas turbines. Figure 1 shows the combined capacities of different types of power generating plants in the four countries of the Nordic electricity market [34].
Figure 1: Installed capacity by production types in Nordic countries on 31 December 2006. Source: Nordel 2006.

The market structure in electricity generation in the Nordic area varies from country to country. Four biggest companies (Vattenfall, Fortum, EON and Statkraft) own over half of the total generation capacity in the Nordic region. Because of market concentration the biggest companies are able to exercise some market power and influence the price of electricity [19].

3 Economic background

There are numerous different market structures present in real world markets. These can be roughly grouped by the number of participants and type of interactions between the participants into following categories: perfect competition, monopolistic competition, oligopoly, oligopsny, monopoly and monopsony [26]. The number of participants varies from the numerous small firms in perfect competition to the single firm supplying the whole market in monopoly. In this study two market structures are salient: perfect competition and oligopoly.

3.1 Perfect competition

A market is said to be perfectly competitive (or purely competitive) if each firm assumes that the market price is independent of its own level of output [26]. Therefore, in a perfectly competitive market firms can focus only on
the level of output they want to produce: everything is sold at the going market price. Firms in a perfectly competitive market are said to be price takers.

The default conditions of the market that give rise to price taking behavior are: a large number of firms producing identical product so that each firm is only a small part of the market. An example of such market would be the market for wheat. Within a domestic economy of, say, a country like the USA, there are thousands of farmers, and even the largest one of them produces a very small portion of the total market supply. It is reasonable therefore for any one farmer in the market to take the market price as fixed. If a farmer wants to sell any of its wheat, it has to sell it at the market price. Other examples where market structure is nearly perfectly competitive are large stock exchanges and eBay auctions. Furthermore, within any market structure there can be small firms that act in perfectly competitive manner (take market price as fixed) even though the structure of the market cannot be described as perfectly competitive.

3.2 Oligopoly

An oligopoly is a market structure where there are several competitors in the market, but not so many as to regard each of them having a negligible effect on price. Oligopoly can give rise to many different strategic interactions depending on the detailed characteristics market and therefore no single model is sufficient to describe different behavior patterns that can be observed in the real world [26]. Because of the strategic nature of interactions between firms in an oligopolistic market structure, game theory is often employed to model the behavior of firms.

The concept of a game theoretic equilibrium as a strategy profile (set of strategies) in which the strategies of different players are consistent with each other is important to the economic analysis of oligopolies. A common type of equilibrium is the Nash-equilibrium defined the strategy profile \( x^* = (x_1^*, ..., x_n^*) \) in which no unilateral deviation in strategy by any player is profitable [22]:

\[
\forall i : f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*),
\]

where \( i \) is the index representing the players, \( f_i \) is the payoff function, \( x_i \) is the strategy of player \( i \) and \( x_{-i} \) are the strategies of players other than \( i \).

Games can be divided into simultaneous and sequential games: in simultaneous games players choose their actions without knowing what others have chosen whereas in sequential games later players have some information about the earlier moves by other players [22]. In this study two applications of simultaneous games in to oligopolistic market structure are of interest:
Bertrand competition and Cournot competition.

### 3.2.1 Bertrand competition

In Bertrand competition, the strategic decision variable of the firms is price. When choosing a price firms try to forecast how the demand for their product depends on price. Firms attempt to choose a price that maximizes their profit. Prices are set simultaneously and the market demand determines the quantity sold for each firm. If firms are selling identical products, the Nash equilibrium in the case of Bertrand equilibrium turns out to be the competitive equilibrium (same as in perfect competition) where price equals marginal cost \([26]\).

### 3.2.2 Cournot competition

In Cournot competition, the strategic decision variable of the firms is the quantity they produce. Each firm has to forecast what other firm’s output will be in order to make a sensible decision itself. After the firms have made their supply decisions the price level is determined by interaction of aggregate supply and demand. In Cournot competition, firms are assumed to take the output of other firms as fixed, that is, the firms assume that the output of other firms is not affected by their decision. This implies the condition:

\[
\forall i : \frac{\partial Q_{-i}}{\partial q_i} = 0, \tag{2}
\]

where \(q_i\) is the output of player \(i\) and \(Q_{-i} = \sum_k q_k - q_i\) is the combined output of firms other than \(i\). Nash equilibrium in the case of Cournot competition (the Cournot equilibrium) is a set of output quantities from which it is not profitable for any player to unilaterally deviate.

### 4 Complementarity problems

Complementarity problems arise in numerous areas of optimization. An important reason why complementarity problems are common is that the concept of complementarity is synonymous with the notion of system equilibrium. The balance of supply and demand (economic equilibrium) is central to many economic models. Also in engineering applications one is often interested to find the equilibria for a system. Furthermore, complementarity problems can be used to model the Karush-Kuhn-Tucker (KKT) optimality conditions for nonlinear programs \([14]\). Examples of areas of optimization with complementarity formulations include structural mechanics problems,
traffic equilibrium problems, network design problems, general equilibrium problems and game-theoretic models [12].

A common class of complementarity problems consists of nonlinear complementarity problems (NCP). A nonlinear complementarity problem is a problem to find a vector \( x \in \mathbb{R}^n \) satisfying the system of equations and inequalities:

\[
F_i(x) \geq 0, \quad x_i \geq 0, \quad x_i F_i(x) = 0 \quad \text{for } i = 1, \ldots, n. \tag{3}
\]

or equivalently,

\[
F(x) \geq 0 \perp x \geq 0, \tag{4}
\]

where \( F : X \rightarrow \mathbb{R}^n \) is a function defined on subset \( X \subseteq \mathbb{R}^n \) containing at least the nonnegative orthant [13].

In addition to NCPs, there are many other types of complementarity problems. The one used in this paper is mixed complementarity problem (MCP) which allows for some variables to be bounded while others are completely free. A mixed complementarity problem is a problem to find a vector \( x \in [l, u] \) such that exactly one of the following holds for all \( i = 1, \ldots, n \) [13]:

\[
\begin{align*}
F_i(x) &> 0 \quad \text{and} \quad x_i = l_i, \\
F_i(x) &< 0 \quad \text{and} \quad x_i = u_i, \\
F_i(x) &= 0 \quad \text{and} \quad x_i \in [l_i, u_i].
\end{align*}
\]

The bounds \( l_i \) and \( u_i \) can be infinite. When \( l_i = -\infty \) and \( u_i = \infty \), the MCP is exactly the problem of finding a zero of \( F \). If \( l_i = 0 \) and \( u_i = \infty \), problem is reduced in to a nonlinear complementarity problem and therefore MCP is a generalization of NCP. On the other hand it is known that with appropriate changes any MCP can be reduced to a larger dimensional NCP [13].

5 The model of electricity market

The model studied in this paper is an oligopolistic model of electricity markets where the firms compete on quantities supplied to different regions. The model is based on a electricity market model by Sulamaa [25]. The time period of the model is one year. The quantities supplied by firms and all other variables in the model are yearly variables.

There are \( R \) regions in the model each containing \( F_r \) firms (where \( r \) is the index of the region). Firms can generate electricity using \( I \) different generation methods (e.g. hydro power, nuclear power). Firms can supply the
electricity to any of the regions \( r \) given there is enough transmission capacity. The problem facing the firms is to choose optimal production mix and optimal supplies to each of the regions.

Firms in a region are assumed to be identical. This means that they are assumed to have the same generation capacities for each generation type \( i \), the same expectations of the behavior of other firms and the same supply requirement to the area outside the regions included in the model. As a result of the assumed identity of firms, the model can be formulated with respect to representative firms of different regions. The behavior of all firms within a region is the same as the behavior of the representative firm.

### 5.1 The optimal generation mix (cost minimization)

A representative firm in region \( r \) faces the problem of choosing an optimal way to generate a given amount of electricity \( U_r \). A firm has \( I \) different ways to generate electricity. The generation of the representative firm by capacity type \( i \) is \( X_{ri} \). Thus, the condition for the required generation of the firm can be written:

\[
U_r \leq \sum_{i=1}^{I} X_{ri}. \tag{5}
\]

The capacity of generation type \( i \) of the representative firm in region \( r \) is \( K_{ri} \). The firm can generate at maximum \( K_{ri} \) units of electricity by using generation type \( i \) (e.g. hydro power). This can be written:

\[
X_{ri} \leq K_{ri}. \tag{6}
\]

The marginal cost of generation is assumed to be constant \( c_i \) for each generation type. The cost function of the firm is then:

\[
C_r(U_r) = \sum_{i=1}^{I} c_i X_{ri}. \tag{7}
\]

Now, the minimization problem for the representative firm can be written as:

\[
\begin{align*}
\text{min} & \quad \sum_{i=1}^{I} c_i X_{ri} \\
\text{s.t.} & \quad X_{ri} \leq K_{ri} \\
& \quad U_r \leq \sum_{i=1}^{I} X_{ri} \\
& \quad X_{ri} \geq 0.
\end{align*}
\]
The first order conditions for the optimum can be derived by using Lagrange’s method [11]. The conditions form a linear complementarity problem:

\[ \forall i = 1, \ldots, I : \]
\[ c_i + \lambda_i - \mu_r \geq 0 \perp X_{ri} \geq 0 \]
\[ K_{ri} - X_{ri} \geq 0 \perp r_{ri} \geq 0 \]
\[ \sum_{k=1}^{I} X_{rk} - U_r \geq 0 \perp \mu_r \geq 0, \]

where \( \lambda_{ri} \) is the shadow price of capacity \( i \) and \( \mu_r \) is the marginal cost of production for the representative firm.

5.2 The optimal supply to regions (profit maximization)

In the previous section, the conditions for the cost minimizing way of generating \( U_r \) units of electricity were derived. The problems that remain are: how much electricity a firm should produce and how much should it supply to each region to maximize its profit? The decision variables of the representative firm located in region \( r \) are the supplies \( S_{rm} \) to each region \( m \). The total amount of electricity that the firm needs to produce (\( U_r \)) to be able to \( S_{rm} \) supply each region \( m \) and \( OUT_r \) to area outside the regions of the model can be written as:

\[ U_r = \sum_{k=1}^{R} S_{rk} + OUT_r. \]  \hspace{1cm} (8)

To decide the amount of electricity to be supplied to each region, a firm needs to have some knowledge about the behavior of consumers in different regions. The behavior of consumers is described by the demand function which gives the amount of electricity demanded at each price of electricity. Here, we assume that the demand function of each area \( k \) has a constant price elasticity of demand \( \epsilon_k \). This leads to the following form of the inverse demand function (inverse demand function gives the maximum price at which the consumers are willing to buy \( Q_k \) units of electricity):

\[ P_k(Q_k) = P_{k0} \left( \frac{Q_k}{Q_{k0}} \right)^{(1/\epsilon_k)}, \]  \hspace{1cm} (9)

where \( P_k \) is the price of electricity in region \( k \), \( Q_k \) is the quantity of electricity demanded, \( P_{k0} \) is the price of electricity in the reference year and \( Q_{k0} \) is the quantity of electricity demanded in the reference year.
At equilibrium, the quantity of electricity demanded by the consumers in region $k$ equals the quantity supplied to that region. The quantity supplied to a region equals the quantity supplied by the firms plus the quantity supplied from the area outside the regions of the model. This can be written as:

$$Q_k = \sum_{l=1}^{R} F_l S_{lk} + I N_k,$$

where $F_r$ is the number of firms in region $r$, $S_{rk}$ is the supply of the representative firm located in region $r$ to region $k$ and $I N_k$ is the amount of electricity supplied from outside to the region $k$.

The total supply from a region to another is limited by the available transmission capacity. The maximum amount of electricity that can be supplied from region $r$ to region $m$ is $SCAP_{rm}$. The transmission capacity constrain can be written:

$$F_r S_{rk} \leq SCAP_{rk}.$$

The goal of the firms is assumed to be profit maximization. The profit of a firm consists of the revenue from sales of electricity to different regions, on the one hand, and of costs of production and transmission, on the other hand. The costs of production are described by the cost function of the previous section. The costs of transmission are caused by transmission tariffs $T_{rk}$ (fee per unit of electricity transmitted) which are assumed to be constant. Therefore, the profit function of the representative firm is:

$$\pi_r(S_{r1}, ..., S_{rR}) = \sum_{k=1}^{R} P_k S_{rk} - C_r(U_r) - \sum_{k=1}^{R} T_{rk} S_{rk}.$$

The profit maximization problem facing the representative firm in region $r$ can be now stated as:

$$\max_{S_{r1}, ..., S_{rR}} \sum_{k=1}^{R} P_k S_{rk} - C_r(U_r) - \sum_{k=1}^{R} T_{rk} S_{rk}
\text{s.t.}
Q_m = P_m(Q_m)(1/\epsilon_m)
Q_m = \sum_{k=1}^{R} F_k S_{km} + I N_m
U_r = \sum_{k=1}^{R} S_{rk} + OUT_r
F(r) S_{rm} \leq SCAP_{rm}
S_{rm} \geq 0.$$
The first order conditions for the optimal solution can be derived by using Lagrange’s method [11]. The first order conditions form a mixed complementarity problem:

\[ \forall m = 1, \ldots, R : \]
\[ -\frac{P_m - T_{rm} - \mu_r - \alpha_{rm}}{P_m} - \frac{SH_{rm}(1 + \omega_{rm})}{\epsilon_m} \geq 0 \quad \perp \quad S_{rm} \geq 0 \]
\[ SCAP_{rm} - F(r)S_{rm} \geq 0 \quad \perp \quad \alpha_{rm} \geq 0, \]

where \( \alpha_{rm} \) is the shadow price of transmission capacity from region \( r \) to region \( m \), \( SH_{rm} \) is the market share of the representative firm in region \( m \) and \( \omega_{rm} = \frac{\partial(Q_m - S_{rm})}{\partial S_{rm}} \) is conjectural variation (a constant parameter of the model), which describes how the firm assumes the supply of other firms will react to a change in the firm’s own supply.

5.3 Market equilibrium

In the previous sections, conditions for profit maximization and cost minimization for firms were derived. These conditions can be put together to calculate the Nash equilibrium of the electricity market: a set of quantities supplied by representative firms to different regions \((S_{r1}, \ldots, S_{rR})_{r=1}^R\) such that no firm can profit by unilaterally changing any of it’s supplies \( S_{rm} \). That is, the conditions for the Nash equilibrium are that every firm maximizes its profit (and minimizes its generation costs) given other firms’ supplies (mixed complementarity problem):

\[ \forall r = 1, \ldots, R; m = 1, \ldots, R; i = 1, \ldots, I : \]
\[ P_m(Q_m) = P_{m0}((Q_m/Q_{m0})^{1/\epsilon_m}) \]
\[ Q_m = \sum_{k=1}^{R} F_k S_{km} + IN_m \]
\[ U_r = \sum_{k=1}^{R} S_{rk} + OUT_r \]
\[ SH_{rm} = \frac{S_{rm}}{Q_m} \]
\[ c_i + \lambda_{ri} - \mu_r \geq 0 \quad \perp \quad X_{ri} \geq 0 \]
\[ K_{ri} - X_{ri} \geq 0 \quad \perp \quad \lambda_{ri} \geq 0 \]
\[ \sum_{i=1}^{I} X_{ri} - U_r \geq 0 \quad \perp \quad \mu_r \geq 0 \]
\[ -\frac{P_m - T_{rm} - \mu_r - \alpha_{rm}}{P_m} - \frac{SH_{rm}(1 + \omega_{rm})}{\epsilon_m} \geq 0 \quad \perp \quad S_{rm} \geq 0 \]
SCAP_{rm} - F(r)S_{rm} \geq 0 \perp \alpha_{rm} \geq 0.

6 Scenario studies

The model was implemented by using General Algebraic Modeling System (GAMS). PATH solver was used to solve the mixed complementarity problems.

Data for the Nordic electricity market was collected mainly from Organisation for the Nordic Transmission System Operators (Nordel) annual statistics for year 2006 [34]. Other sources were Nord Pool [33], European Transmission System Operators (ETSO) [29] and International Energy Agency [23]. A review of data used in the model is shown in appendix A.

Electricity market equilibria were calculated for several scenarios. The scenarios studied were: equal number \( F \) of Cournot players in each country, competitive equilibrium, number of Cournot firms in each country in 2006, market equilibrium in absence of transmission capacity limits and effect of a carbon price.

6.1 Equal number of Cournot players in each country

This scenario assumes that the number of (identical) firms in each country is the same and each of them acts a Cournot player: the firms assume that changes in its own supply do not affect the supply of other firms. Cournot behavior is modeled by setting the conjectural variations of each firm to zero:

\[
\omega_{rm} = \frac{\partial(Q_m - S_{rm})}{\partial S_{rm}} = 0.
\]

The generation capacity in each country was divided evenly among the \( F \) firms according to the identity assumption. Market price and quantity of electricity supplied was calculated for each country. Table 1 and table 2 show the results for \( F = 1, 2, 3, 4, 5, 10, 100, 10000 \) firms in each country. The real quantity supplied and the price level in each country in 2006 are also included in the tables [34].

Tables 1 and 2 show the trend that the more firms in each country, the lower the price level and higher the quantity supplied. This is because the smaller the number of firms the greater the market power of the firms in each country and the more the firms will be able to drive up price by restricting output. In the case of one Cournot firm in each country, the situation is
Table 1: Price level (EUR/MWh) in each country in case of different number of firms. Values for reference year 2006 are also shown.

<table>
<thead>
<tr>
<th>Number of firms (F)</th>
<th>Finland</th>
<th>Sweden</th>
<th>Norway</th>
<th>Denmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>223.55</td>
<td>101.11</td>
<td>681.04</td>
<td>70.24</td>
</tr>
<tr>
<td>2</td>
<td>73.49</td>
<td>37.89</td>
<td>80.01</td>
<td>30.92</td>
</tr>
<tr>
<td>3</td>
<td>56.19</td>
<td>34.94</td>
<td>33.74</td>
<td>33.18</td>
</tr>
<tr>
<td>4</td>
<td>44.51</td>
<td>31.42</td>
<td>33.74</td>
<td>30.71</td>
</tr>
<tr>
<td>5</td>
<td>41.10</td>
<td>32.36</td>
<td>33.21</td>
<td>31.66</td>
</tr>
<tr>
<td>10</td>
<td>32.86</td>
<td>32.24</td>
<td>32.43</td>
<td>32.12</td>
</tr>
<tr>
<td>100</td>
<td>30.71</td>
<td>27.68</td>
<td>30.38</td>
<td>25.96</td>
</tr>
<tr>
<td>10000</td>
<td>30.71</td>
<td>27.01</td>
<td>30.54</td>
<td>25.54</td>
</tr>
<tr>
<td>Year 2006</td>
<td>48.57</td>
<td>48.12</td>
<td>49.10</td>
<td>46.36</td>
</tr>
</tbody>
</table>

Table 2: Quantity supplied (TWh) in each country in case of different number of firms. Values for reference year 2006 are shown also.

<table>
<thead>
<tr>
<th>Number of firms (F)</th>
<th>Finland</th>
<th>Sweden</th>
<th>Norway</th>
<th>Denmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48.86</td>
<td>108.76</td>
<td>42.81</td>
<td>30.82</td>
</tr>
<tr>
<td>2</td>
<td>76.25</td>
<td>161.04</td>
<td>135.80</td>
<td>42.79</td>
</tr>
<tr>
<td>3</td>
<td>84.90</td>
<td>166.35</td>
<td>142.43</td>
<td>41.60</td>
</tr>
<tr>
<td>4</td>
<td>93.19</td>
<td>173.57</td>
<td>142.43</td>
<td>42.91</td>
</tr>
<tr>
<td>5</td>
<td>96.21</td>
<td>171.54</td>
<td>143.33</td>
<td>42.39</td>
</tr>
<tr>
<td>10</td>
<td>105.22</td>
<td>171.79</td>
<td>144.70</td>
<td>42.15</td>
</tr>
<tr>
<td>100</td>
<td>108.11</td>
<td>182.60</td>
<td>148.52</td>
<td>45.89</td>
</tr>
<tr>
<td>10000</td>
<td>108.11</td>
<td>184.41</td>
<td>148.22</td>
<td>46.19</td>
</tr>
<tr>
<td>Year 2006</td>
<td>89.99</td>
<td>146.37</td>
<td>122.57</td>
<td>36.39</td>
</tr>
</tbody>
</table>

near monopoly. It is not monopoly because firms restrict each others’ market power by exporting to other countries. However, the transmission capacity between countries is limited and only a fraction of the total demand can be imported from other countries. Thus, the degree of market power of each firm in its domestic market is great and the price level with one Cournot firm in each country is significantly higher than in year 2006.

As the number of firms in each country continues to increase the price levels decrease and quantities supplied increase. The price levels and quantities
supplied seem to converge toward some value since there is not much change from having 100 firms and having 10000 firms in each country. However, there are few instances where the price level in one country actually increases as the number of firms increases. This is because the number of firms and the degree of competition has increased in every country, not just in one country. As the degree of competition in domestic markets increases, firms can no longer restrict the supply in domestic market so much. They will supply more to the domestic market. In some cases this means cutting exports to other countries. The combined decrease in exports to a country can be greater than increase in domestic supply. Therefore, it is possible that prices rise in some countries as \( F \) grows. However, as seen from tables 1 and 2 this is a temporary phenomenon.

There are differences between the price levels in each country. This reflects differences in demand, available generation capacity, type of generation capacity, transmission capacities in and out of each country and quantity of exports and imports to/from outside the model. It seems that the most important determinants of the price differences are the amount of demand in relation to available inward transmission capacity and the imports from outside the model. Denmark is the most open market in the sense that the ratio of yearly inward transmission capacity to yearly demand is the greatest. The second most open is Sweden, third is Norway and fourth is Finland [34]. One would expect that the more open the market the lower the price level. This is indeed the main pattern in table 1. However, this pattern is not all that is observed in table 1 because there are other factors affecting the price. Most importantly, when \( F \) is small and firms are more able to restrict output, imports from outside the model area affect the price level significantly. This explains why with \( F = 1 \) Finland does not have the highest price level: about 11.5TWh of Russian imports keep the price down.

### 6.2 Competitive equilibrium

In perfect competition, firms are price takers and assume they have no influence on price. equilibrium in perfectly competitive markets is called competitive equilibrium. In competitive equilibrium, prices in each country equal the marginal cost of supply (production + transmission). By setting \( \omega_{rm} = -1 \) all firms will act in price taking manner because the profit maximization criteria in MCP reduces to

\[
\frac{P_m - T_{rm} - \mu_r - \alpha_{rm}}{P_m} \leq 0 \quad S_{rm} \geq 0
\]  

That is, if a firm supplies a positive amount \( S_{rm} \) then price will have to equal marginal cost \( T_{rm} + \mu_r + \alpha_{rm} \). It doesn’t matter how many firms...
operate in each country because the resulting equilibrium will be the same. If \( \omega_{rm} = -1 \), all capacity will act competitively.

Table 3 shows the price level and quantity supplied in the case of each country in competitive equilibrium.

Table 3: Price level and quantity supplied in competitive equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>Finland</th>
<th>Sweden</th>
<th>Norway</th>
<th>Denmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price level (EUR/MWh)</td>
<td>30.71</td>
<td>27.00</td>
<td>30.54</td>
<td>25.54</td>
</tr>
<tr>
<td>Quantity supplied (TWh)</td>
<td>108.11</td>
<td>184.43</td>
<td>148.22</td>
<td>46.19</td>
</tr>
<tr>
<td>Price in year 2006</td>
<td>48.57</td>
<td>48.12</td>
<td>49.10</td>
<td>46.36</td>
</tr>
<tr>
<td>Quantity in year 2006</td>
<td>89.99</td>
<td>146.37</td>
<td>122.57</td>
<td>36.39</td>
</tr>
</tbody>
</table>

By comparing tables 1, 2 and 3 it can be seen that the competitive equilibrium matches (only second decimal in case of Sweden differs) the equilibrium with 10000 Cournot firms. This result was expected because competitive equilibrium is a result of very large number of small firms acting in the same market.

In the competitive equilibrium prices are significantly lower than in year 2006. The price is lowered most significantly in the case of Denmark and least in the case of Finland and Norway. This indicates that currently the level of market power exercised by generating firms in Denmark is greater than in Finland and Norway. A transition from current market structure to a perfectly competitive one would cut prices almost by half. This seems to indicate that the power companies operating in the Nordic electricity market are able to exercise considerable amount of market power.

6.3 Number of Cournot firms \( F_r \) in 2006

By fixing the quantities supplied to each country \( Q_r \) to their reference year levels it is possible to determine the number of Cournot firms \( (\omega_{rm} = 0) \) \( F_r \) in each country that would produce such an equilibrium. When doing this the number of firms \( F_r \) is assumed to be a real number instead of whole number. Table 4 shows the number of Cournot firms that replicate the 2006 equilibrium.

As number of firms \( F_r \) is a real number here, the results cannot be taken literally: a fraction of a firm cannot exist. However, the results can be interpreted to mean that the number of firms is at some point between two whole numbers or that the market contains certain number of equal sized
The number of Cournot firms is the greatest in Finland (3.74) and the smallest in Denmark (0.91). This indicates that the degree of domestic competition (reflected by the number of identical firms) is greatest in Finland and the smallest in Denmark with Sweden and Norway in between. The results seem indicate that the national markets are very concentrated (with perhaps the exception of Finland). The fact is in the real world the number of firms is greater [28]. The exact numbers $F_r$ should not be taken literally. Despite that the order of magnitudes can give useful information and the values can be used for calibrating the model to replicate the year 2006 market equilibrium.

### 6.4 Equilibrium in 2006 in the absence of transmission limits

By using the results of the previous section it is possible to estimate the effect of removal of all transmission capacity limits. This corresponds to the case that a huge amount of transmission capacity would be suddenly added between all countries in the Nordic electricity market. In the model this means setting $SCAP_{rm} = \infty$ and fixing $F_r$ to values shown in table 4. Table 5 shows the price level and quantity supplied in each country when transmission capacity limits are removed.

Table 5: Price level and quantity supplied in each country in 2006 if all transmission capacity limits are removed.

<table>
<thead>
<tr>
<th></th>
<th>Finland</th>
<th>Sweden</th>
<th>Norway</th>
<th>Denmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price level (EUR/MWh)</td>
<td>33.03</td>
<td>36.13</td>
<td>36.23</td>
<td>35.58</td>
</tr>
<tr>
<td>Quantity supplied (TWh)</td>
<td>105.00</td>
<td>164.15</td>
<td>138.43</td>
<td>40.45</td>
</tr>
<tr>
<td>Price in year 2006</td>
<td>48.57</td>
<td>48.12</td>
<td>49.10</td>
<td>46.36</td>
</tr>
<tr>
<td>Quantity in year 2006</td>
<td>89.99</td>
<td>146.37</td>
<td>122.57</td>
<td>36.39</td>
</tr>
</tbody>
</table>
The price level in each country is lowered significantly as a result of removal of transmission capacity limits. This is due to increase in the degree of competition in each country as foreign firms are more able to compete with domestic ones. The price levels are decreased by approximately the same amount in every country, except in Finland where the decrease in price is a bit larger. This can be seen as a result of the fact that initially Finland was the most closed market (the ratio of yearly inward transmission capacity to yearly demand was the smallest). Thus, one would expect the removal of transmission capacity limits to lower prices in Finland the most. Another possible reason behind the lower Finnish price is that Finland has the highest degree of domestic competition (measured by the number of firms). Furthermore, Finland is the biggest net importer of electricity from countries outside the model which also contributes to a lower price.

6.5 Effect of a carbon price

This scenario studies hypothetically what would happen if there were an uniform carbon price in the Nordic region. The carbon price affects the generating cost of carbon producing types of plants (CHP, condensing and gas turbines) but does not affect carbon-free plants (hydro power, wind power and nuclear power). In this scenario following increases in values for marginal generating costs are assumed: hydro power (0), wind power (0), nuclear power (0), CHP (5 EUR/MWh), condensing (8 EUR/MWh) and gas turbines (12 EUR/MWh). These values are based on a study by Reinaud (2005) which estimates the effect of 20 EUR/ton of CO\textsubscript{2} carbon price on generating costs of different types of capacities [23]. The values are hypothetical because the plant types used by Reinaud do not exactly match the plant types used in this study.

To study the effect of the carbon price, \( F_r \) was fixed to the values in table 4 and the generating costs for plant types were changed to account for the carbon price. Table 6 shows the resulting equilibrium.

Table 6: Price level and quantity supplied in each country after a hypothetical carbon price (20 EUR/ton of \( CO_2 \)).

<table>
<thead>
<tr>
<th></th>
<th>Finland</th>
<th>Sweden</th>
<th>Norway</th>
<th>Denmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price level (EUR/MWh)</td>
<td>56.19</td>
<td>48.27</td>
<td>49.10</td>
<td>50.05</td>
</tr>
<tr>
<td>Quantity supplied (TWh)</td>
<td>84.90</td>
<td>146.19</td>
<td>122.57</td>
<td>35.29</td>
</tr>
<tr>
<td>Price in year 2006</td>
<td>48.57</td>
<td>48.12</td>
<td>49.10</td>
<td>46.36</td>
</tr>
<tr>
<td>Quantity in year 2006</td>
<td>89.99</td>
<td>146.37</td>
<td>122.57</td>
<td>36.39</td>
</tr>
</tbody>
</table>
As a result of the carbon price, price of electricity rises and quantity of electricity supplied decreases in most cases. In the case of Norway the price and quantity supplied stays exactly the same. The increase in price in the case of Sweden is very small. This is because Norway and Sweden have high amounts of carbon-free generation capacity (hydro power in Norway and hydro power and nuclear power in Sweden). The carbon price does not affect them much because they are using carbon-free generation plants. The price rise is significant in Finland (8 EUR/MWh) and Denmark (4 EUR/MWh). These countries use the plant types which generate carbon in the initial 2006 equilibrium (CHP and condensing plants for Finland and CHP for Denmark). The carbon price affects the generation costs of these plants which increases the price of electricity in these markets.

7 Conclusion

This paper presented a conjectural variation model of electricity markets. The model assumes identity of firms within regions (same expectations and equal capacity). The model has exogenous imports and export requirements outside the model area and the number of firms in each region is formulated as an endogenous variable. The model was used to analyse some scenarios in the Nordic electricity market based mainly on data from Nordel 2006 annual statistics.

The model performed fairly well in the various scenario studies investigated. The results were all in all reasonable and made sense qualitatively. Quantitatively, the results are sensitive to various model parameters which are uncertain. Therefore, one should not attempt to analyse the results quantitatively.

The main results were: the price of electricity goes down and supply of electricity goes up as the number of firms in each area increases, equilibrium with very high number of Cournot firms is the same as competitive equilibrium, if Nordic electricity markets were perfectly competitive, price would be cut almost by half, the competition among domestic firms is highest in Finland and lowest in Denmark with Sweden and Norway in between, removing all transmission capacity limits cuts the price of electricity significantly and an uniform carbon price increases prices in regions where carbon producing generation capacity is used. These results seem qualitatively reasonable, although there is not much comparative data available for cross-checking the results.

One case where there is some comparative data available is the degree of competition within national markets. Nordic competition authorities calculated the concentration indexes (Herfindahl-Hirschman Index) for electricity
generation industry in 2003 [28]. These indexes show that degree of concentration in 2003 is highest in Denmark and lowest in Norway (with Finland closely behind). The results from the model also identified Denmark as having the lowest degree of domestic competition but identified Finland as having clearly the highest degree of domestic competition. One reason why the model estimated that Finland has the most competitive domestic economy is that Finnish market was the most closed (in terms of ratio of inward transmission capacity from Nordic countries to demand). The concentration indexes from 2003 are not a very good comparison point for results derived from 2006 data but they give some indication. It is also notable that concentration is not same as the degree of competition. Furthermore, there are serious problems with using concentration indexes to measure the extent of market power as indicated in [7].

There are some shortcomings to the model. One is that conjectural variations models and Cournot models require that the market share of each firm is smaller than the absolute value of price elasticity of demand. Otherwise the solution is not feasible. This condition can be easily derived from the complementarity conditions for supply to different regions $S_{rm}$. This requirement limits the range of possible scenarios that can be investigated with the model. On the one hand situations where the price elasticity demand is very small are infeasible and on the other hand situations with firms having very high market share are infeasible. This problem is also identified for example in [25, 10].

The time period of the model is one year. This far too large time horizon to be able to realistically model the electricity markets. It is characteristic of the electricity market to have large fluctuations in demand and price of electricity within one year. Demand varies from month to month according to the time of the year (more demand in winter). The accumulated water reservoirs affect significantly the price of electricity because hydro power is so prevalent in the Nordic countries. There are systematic variations in demand and price even within one day. For example, for analysis of the market power in electricity markets, it is essential to investigate the periods of peak demand when market power can manifest itself. It is not possible to do this with the model studied. To cope with this problem, one could change the time period of the model to, for example, a month. This should be fairly straightforward. However, the problem is then to obtain monthly data of each the required variables (e.g. demand).

One unrealistic feature that often manifests itself with Cournot type oligopoly models is the phenomenon of reciprocal dumping. Reciprocal dumping means that there is seemingly unnecessary two-way trade in the same product between two countries. That is, there is trade between countries far in excess of the difference in domestic demand and the capacity for domestic
supply. This phenomenon was also observed in the model studied. In most of scenarios investigated there was maximum amount of trade allowed by transmission capacity limits and the shadow price of additional transmission capacity was high. It is unclear whether this kind of behavior is observed in the real world. In any case it is doubtful that the extent of the phenomenon is as large in the real world as it seems to be in the model. reciprocal dumping is extensively discussed by Brander and Krugman (1983). They conclude that the welfare effects of such trade are ambiguous: on the one hand resources are wasted in the cross-handling of goods; on the other hand, increased competition reduces monopoly distortions [8].

The model was designed with an eye on the available data. The annual statistics for each country could be easily obtained from Nordel. It is much harder to obtain firm level data because firms are reluctant to publish strategic values such as available capacities. If firm or plant level data were available, the identity assumption of the model could be dropped and the model could be easily changed to account for such data. In addition to more detailed description of the industries in countries, the model could be made better by choosing a shorter time period such as a month. In shorter time periods essential fluctuations such as the changes in water reservoirs could be included in the model. Furthermore, trade with countries outside the model could be modeled with greater precision (e.g. with dependence to price) instead of fixed exogenous imports and exports. In general, it can be said that the modeling of multinational electricity market is a big task with many different aspects that need to be taken into account. No simple model can account for the multitude of forces, ranging from emission trade to water reservoir management, that are essential features of the Nordic electricity market.

References


A Review of the data used


<table>
<thead>
<tr>
<th></th>
<th>Hydro</th>
<th>Wind</th>
<th>Nuclear</th>
<th>CHP</th>
<th>Condensing</th>
<th>Gasturbines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finland</td>
<td>3044</td>
<td>86</td>
<td>2671</td>
<td>6661</td>
<td>3301</td>
<td>781</td>
</tr>
<tr>
<td>Sweden</td>
<td>16180</td>
<td>580</td>
<td>8965</td>
<td>4183</td>
<td>2298</td>
<td>1613</td>
</tr>
<tr>
<td>Norway</td>
<td>28691</td>
<td>333</td>
<td>0</td>
<td>180</td>
<td>0</td>
<td>64</td>
</tr>
<tr>
<td>Denmark</td>
<td>10</td>
<td>3135</td>
<td>0</td>
<td>8254</td>
<td>993</td>
<td>307</td>
</tr>
</tbody>
</table>

Table: Yearly maximum load utilization times and marginal generation costs different types of capacities. Sources: Energiateollisuus ry, IEA 2005.

<table>
<thead>
<tr>
<th>Utilization time (hours)</th>
<th>Hydro</th>
<th>Wind</th>
<th>Nuclear</th>
<th>CHP</th>
<th>Condensing</th>
<th>Gastur</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>18</td>
<td>27</td>
<td>36</td>
</tr>
</tbody>
</table>

Table: Price level, consumption, imports from other countries and exports to other countries in 2006 in the case of each Nordic country. Here ‘other countries’ means countries outside the model area. Sources: Nord Pool, Nordel 2006.

<table>
<thead>
<tr>
<th></th>
<th>Finland</th>
<th>Sweden</th>
<th>Norway</th>
<th>Denmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price level (EUR/MWh)</td>
<td>48.57</td>
<td>48.12</td>
<td>49.10</td>
<td>46.36</td>
</tr>
<tr>
<td>Consumption (TWh)</td>
<td>89.99</td>
<td>146.37</td>
<td>122.57</td>
<td>36.39</td>
</tr>
<tr>
<td>Imports from other countries (TWh)</td>
<td>11.55</td>
<td>3.40</td>
<td>0.22</td>
<td>3.96</td>
</tr>
<tr>
<td>Exports to other countries (TWh)</td>
<td>0.01</td>
<td>1.78</td>
<td>0</td>
<td>5.84</td>
</tr>
</tbody>
</table>

Table: Transmission capacity between countries (MW). Source: Nordel 2006.

<table>
<thead>
<tr>
<th></th>
<th>Finland</th>
<th>Sweden</th>
<th>Norway</th>
<th>Denmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finland</td>
<td>-</td>
<td>1830</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Sweden</td>
<td>2230</td>
<td>-</td>
<td>3340</td>
<td>2240</td>
</tr>
<tr>
<td>Norway</td>
<td>100</td>
<td>3620</td>
<td>-</td>
<td>1000</td>
</tr>
<tr>
<td>Denmark</td>
<td>0</td>
<td>2820</td>
<td>1000</td>
<td>-</td>
</tr>
</tbody>
</table>

Yearly utilization time of transmission capacity was assumed to be 8000 hours. Price elasticity of demand was assumed to be -0.4 in every country (Source: Sulamaa, Essays in Deregulated Finnish and Nordic Electricity Markets). Transmission cost from a country to another was assumed to be
5 EUR/MWh (Source: ETSO Overview of transmission tariffs in Europe: Synthesis 2006). Transmission within a country was assumed to be costless.