Support for Investment Planning with Contingent Portfolio Programming

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Issues in project portfolio selection

1. Limited resources
   - Not all projects can be started

2. Uncertainties
   - Profitability of projects depends on market developments

3. Follow-on decisions
   - Earlier results offer later opportunities

4. Alternative implementation strategies
   - Projects can be carried out in several different ways

4. Project interactions
   - Complementarities entail synergies among projects
   - Some projects may be prerequisites for others
Representative earlier approaches

- **Dynamic programming models**
  - Decision tree formulations (eg Hespos & Strassman 1965)
  - Capture external and project-specific uncertainties
  - Multiple implementation strategies can be readily modeled
  - Deficient in addressing multiple resource constraints
  - Are not tractable for large project portfolios

- **Linear programming models**
  - Accommodate different kinds of resource constraints
  - Project interactions can be captured (eg, variables for joint effects)
  - Typically static capital budgeting models (eg, Gear et al. 1971)
  - Weak in accounting for uncertainties
    - Cf. chance-constrained models, fuzzy sets and utility functions
Contingent Portfolio Programming (CPP)

Captures
- External uncertainties through scenario trees
- Multiple resources, resource constraints and resource dynamics
- Follow-on decision opportunities and project interactions
- DM’s risk attitude

Provides
- Optimal project management strategies
- Valuation framework for mixed asset portfolio selection
  » “extends methods for decision analysis of risky projects to account for the opportunities to invest in financial securities” or, alternatively, “extends the CAPM to permit evaluation of projects that are not financial securities” (Kleinmuntz’s statement on Janne’s Dissertation)

Most Recent Winner(s):

<table>
<thead>
<tr>
<th>Year</th>
<th>Name(s) and Affiliation</th>
<th>Title</th>
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<tbody>
<tr>
<td>2006</td>
<td>Michael Braun, MIT Sloan School of Management</td>
<td>&quot;Modeling the “Pseudodependable” in Insurance Claims Decisions&quot;</td>
</tr>
<tr>
<td>2005</td>
<td>Janne Gustafsson, Helsinki Institute of Technology</td>
<td>“Contingent Portfolio Programming for the Management of Risky Projects”</td>
</tr>
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<td>2004</td>
<td>Warren J. Hahn, University of Texas at Austin</td>
<td>&quot;A Discrete-Time Approach for Valuing Real Options with Underlying Mean-Reverting Stochastic Processes&quot;</td>
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<td>2003</td>
<td>Franz H. Henkamp, IESE Business School, Barcelona</td>
<td>&quot;Stochastic Dominance and Cumulative Prospect Theory: Theory and Experience&quot; with Manel Baucells</td>
</tr>
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<td>2002</td>
<td>Veronika Köberling, Maastricht University</td>
<td>&quot;Preference Foundations for Difference Representations&quot;</td>
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<td>2001</td>
<td>Manel Baucells, IESE Business School, Barcelona</td>
<td>&quot;Multiperson Utility&quot;</td>
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<td>2000</td>
<td>Cade Massey, University of Chicago</td>
<td>&quot;Detecting Regime Shifts: A Study of Over- and Under-Reaction&quot;</td>
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<td>1999</td>
<td>Kara M. Morgan, Carnegie Mellon University</td>
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<tr>
<td>1997</td>
<td>Osvaldo F. Moreira, University of Illinois-Chicago</td>
<td>&quot;A Psychometric Analysis of the 'Divide and Conquer' Principle in Decision Analysis&quot;</td>
</tr>
</tbody>
</table>
An illustrative example

- Two projects (A and B) such that
  - Either project (or both) can be started in period 0
  - Further investment is possible in period 1
  - Uncertain cash flow from sales in period 2

- A single resource
  - Initial budget $b = $90m
  - Interest rate $r = 8\%$

- Key components
  - Scenario tree for modeling of uncertainties
  - Binary decision variables for both projects
  - Maximization of risk-adjusted net cash flow position in period 2
Scenarios and decisions

Scenarios

- $S_0$ (50%)
- $S_1$ (50%)
- $S_2$ (50%)
- $S_{11}$ (30%)
- $S_{12}$ (70%)
- $S_{21}$ (40%)
- $S_{22}$ (60%)

Time period

- 0
- 1
- 2

Projects

- A
  - Start? Yes
  - Continue? Yes
  - Start? No
  - Continue? No

- B
  - Start? Yes
  - Continue? Yes
  - Start? No
  - Continue? No
Combined scenario and decision trees

Project A

Start?

Yes: -$10m

No

Continue?

Yes: -$30m

No

Project B

Continue?

Yes: -$20m

No

Continue?

Yes: -$30m

No
Project-specific cash flows

Project A

- $-30 \times X_{ACY1}$
- $-10 \times X_{ASY}$
- $-30 \times X_{ACY2}$
- $50 \times X_{ACY2}$

Scenarios:
- $S_0$
- $S_1$
- $S_2$
- $S_{11}$
- $S_{12}$
- $S_{21}$
- $S_{22}$

Time period:
- 0
- 1
- 2

Project B

- $-20 \times X_{BCY1}$
- $-20 \times X_{BSY}$
- $-20 \times X_{BCY2}$
- $25 \times X_{BCY1}$
- $10 \times X_{BCY1}$
- $250 \times X_{BCY2}$
- $100 \times X_{BCY2}$

Scenarios:
- $S_0$
- $S_1$
- $S_2$
- $S_{11}$
- $S_{12}$
- $S_{21}$
- $S_{22}$

Time period:
- 0
- 1
- 2
Portfolio cash flows

Maximize risk-adjusted NPV
- Computed in period 2
- in this case
  \[ EV(\mathbf{X}) - \lambda \text{LSAD}(\mathbf{X}) \]
  with \( \text{LSAD} = \) lower semiabsolute deviation from EV
- Results in a linear model

Constraints
- Consistency constraints
- Resource constraints
- Deviation constraints
Basic constraints

Consistency constraints:

- $X_{ASY} + X_{ASN} = 1$
- $X_{ACY1} + X_{ACN1} = X_{ASY}$
- $X_{ACY2} + X_{ACN2} = X_{ASY}$
- $X_{BSY} + X_{BSN} = 1$
- $X_{BCY1} + X_{BCN1} = X_{BSY}$
- $X_{BCY2} + X_{BCN2} = X_{BSY}$

Resource constraints:

- $R_{S0} = 90 - 10 \cdot X_{ASY} - 20 \cdot X_{BSY}$
- $R_{S1} = 1.08 \cdot R_{S0} - 30 \cdot X_{ACY1} - 20 \cdot X_{BCY1}$
- $R_{S2} = 1.08 \cdot R_{S0} - 30 \cdot X_{ACY2} - 20 \cdot X_{BCY2}$
- $R_{S11} = 1.08 \cdot R_{S1} + 200 \cdot X_{ACY1} + 25 \cdot X_{BCY1}$
- $R_{S12} = 1.08 \cdot R_{S1} + 100 \cdot X_{ACY1} + 10 \cdot X_{BCY1}$
- $R_{S21} = 1.08 \cdot R_{S2} + 50 \cdot X_{ACY2} + 250 \cdot X_{BCY2}$
- $R_{S22} = 1.08 \cdot R_{S2} + 100 \cdot X_{BCY2}$

$R_S$’s are resource surplus variables
- indicate the amount of resources at the end of a scenario
Deviation constraints

\[ V_s - EV - \Delta V_s^+ + \Delta V_s^- = 0 \]

defined for each terminal scenario

Deviation constraint for scenario \( s_{11} \):

\[ RS_{s11} - EV - \Delta V_s^+ + \Delta V_s^- = 0 \]

\[ RS_{s11} - \left[ 50\% \cdot 30\% \cdot RS_{s11} + 50\% \cdot 70\% \cdot RS_{s11} + 50\% \cdot 40\% \cdot RS_{s21} + 50\% \cdot 60\% \cdot RS_{s22} \right] \]

\[ - \Delta V_{11}^+ + \Delta V_{11}^- = 0 \]

Deviation variables \( \Delta V_{11}^- \) penalized in the objective function

\[ \Rightarrow \Delta V_{11}^+, \Delta V_{11}^- \text{ cannot both be positive} \]
Objective function

Maximize

$$CE = EV - \lambda \cdot LSAD = \left[ p_{11} \cdot RS_{s11} + p_{12} \cdot RS_{s12} + 
\quad p_{21} \cdot RS_{s21} + p_{22} \cdot RS_{s22} \right]$$
\[ - 0.50 \cdot \left[ p_{11} \cdot \Delta V_{s11}^- + p_{12} \cdot \Delta V_{s12}^- + 
\quad p_{21} \cdot \Delta V_{s21}^- + p_{22} \cdot \Delta V_{s22}^- \right] \]

subject to preceding constraints
**Optimal solution**

Project A

- **Start?**
  - Yes: -$10m
  - No: $0

- **Continue?**
  - Yes: -$30m
  - No: $50m

Project B

- **Start?**
  - Yes: -$20m
  - No: $0

- **Continue?**
  - Yes: -$20m
  - No: $0

**Trends and Developments in Decision Analysis, September 6, 2007**
Observations

- Properties of optimum solution
  - $EV = $188m and $LSAD = $29.5m at period 2 $\Rightarrow CE = $173.3m$
  - Lowest resource surplus in scenario $s_{12}$ with $RS_{12} = $137.6m
    $\Rightarrow$ Optimal portfolio is surely better than depositing the initial budget at 8% interest rate $(1.08)^2 \times $90m = $105.m

- Additional constraints
  - Can be readily captured through constraints on decision variables
  - E.g., “project B cannot be continued unless project A was started”
    $\Rightarrow X_{BCY_1} \leq X_{ASY}$ and $X_{BCY_2} \leq X_{ASY}$
Other risk measures

- **Expected Downside Risk (EDR)**
  - Defined relative to the target level $t$
  - Deviation variables $\Delta V_s^+, \Delta V_s^-$ from $V_s - t - \Delta V_s^+ + \Delta V_s^- = 0$

  $$EDR = \sum_{RS_s < t} p_s \Delta V_s^-$$

- **Absolute Deviation (AD)**
  - Defined relative to the expectation $EV$
  - Deviation variables $\Delta V_s^+, \Delta V_s^-$ from $V_s - EV - \Delta V_s^+ + \Delta V_s^- = 0$

  $$AD = \sum_{s} p_s (\Delta V_s^- + \Delta V_s^+)$$
Mean-risk model with risk constraints

Objective function:

\[
\max_{x,z} W = \sum_s p(s) \cdot RS_s
\]

Deviation constraint:

\[
RS_s - t(RS) - \Delta_s^+ + \Delta_s^- = 0
\]

Domain constraints:

\[z_k \in \{0,1\}, \quad x_i \text{ free}\]

Risk constraint:

\[
\rho(\Delta^-, \Delta^-) \leq R, \quad \Delta_s^+, \Delta_s^- \geq 0
\]

- LSAD \[\sum p(s) \cdot \Delta_s^-\]
- AD \[\sum p(s) \cdot (\Delta_s^+ + \Delta_s^-)\]
Computational performance

<table>
<thead>
<tr>
<th>Problem characteristics</th>
<th>Model size</th>
<th>Computation time</th>
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<tbody>
<tr>
<td></td>
<td>No. of variables</td>
<td>No. of constraints</td>
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<tr>
<td>Projects</td>
<td>Stages</td>
<td>Periods</td>
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</table>
## Alternative Value-At-Risk Measures (1/2)

<table>
<thead>
<tr>
<th>Measures</th>
<th>Use Asset Returns</th>
<th>Use Cash Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measures maximum loss with certain confidence level</td>
<td>VAR (U.S. SEC, ~1980)</td>
<td>CFAR (Linsmeier and Pearson 2000)</td>
</tr>
<tr>
<td>Measures expected loss with certain confidence level conditional tail event occurred</td>
<td>CVAR (Rockafeller and Uryasev 2000 and Artzner et. al. 1999)</td>
<td>CCFAR (this research)</td>
</tr>
</tbody>
</table>
Alternative Value-At-Risk Measures (2/2)

\[ CVAR_\beta(x) = \min_{\alpha} \left( \alpha + \frac{1}{1-\beta} \int_{f(x,y) > \alpha} (f(x,y) - \alpha) p(y) dy \right) \]

(\text{Rockafeller and Uryasev, 2000})

- \( \beta \) = confidence level
- \( f(x,y) \) = loss function
- \( x \) = portfolio
- \( y \) = uncertainty
- \( \alpha \) = threshold value (=VAR)
- \( p(y) \) = probability density function
Conditional-Cash-Flow-At-Risk (CCFAR)

- CCFAR can be derived from CVAR
  - Discrete version
  - Portfolio loss defined in terms of cash flows \( C - C S_\omega \)

\[
CCFAR_\beta (x, z) = \min_{\alpha, \kappa_\omega} \left( \alpha + \frac{1}{1 - \beta} \sum_{\omega \in \Omega} \kappa_\omega \right)
\]

\[\text{s.t.} \quad \kappa_\omega \geq p(\omega) \cdot (C - C S_\omega - \alpha)\]

\[\kappa_\omega \geq 0\]

Definitions:
- \( \alpha \) = threshold value (=VAR in CCFAR calc.)
- \( \beta \) = confidence level
- \( x \) = security portfolio mgmt strategy
- \( z \) = project portfolio mgmt strategy
- \( p(\omega) \) = probability of scenario \( \omega \)
- \( C \) = initial cash
- \( C S_\omega \) = cash surplus in scenario \( \omega \) \( \in \Omega \)
CVAR is a Coherent Risk Measure

- Let X and Y be random returns

- A coherent risk measure (such as CVAR) satisfies the following properties
  - Translation invariance: \( CVAR(x + a) = CVAR(x) - a \quad \forall a \in \mathbb{R} \)
  - Subadditivity: \( CVAR(x + y) \leq CVAR(x) + CVAR(y) \quad \forall x, y \)
  - Positive Homogeneity: \( CVAR(\lambda x) = \lambda CVAR(x) \quad \forall \lambda \geq 0 \)
  - Positivity: \( CVAR(x) \leq 0 \quad \forall x \geq 0 \)
Risk Constraint Matrix (RCM)

\[
M = \begin{pmatrix}
R_{t_1, \beta_1} & \cdots & R_{t_1, \beta_m} \\
\vdots & \ddots & \vdots \\
R_{t_n, \beta_1} & \cdots & R_{t_n, \beta_m}
\end{pmatrix}
\]

Risk measure \(_{i,j} \leq R_{t_i, \beta_j}\)

- **Benefits**
  - A compact representation of risk requirements
  - Supports the mgmt of risks in
    - different time periods
    - different percentiles
  - Admits complementary risk measures

**Definitions:**

\(M \in \mathbb{R}^{n \times m} = \) risk constraint matrix

\(R_{t_n, \beta_m} \in \mathbb{R} = \) risk tolerances at different time periods \(t_1, \ldots, t_n\) and confidence levels \(\beta_1, \ldots, \beta_m\) when \(n, m \in \mathbb{Z}^+\) and \(n \leq T\)

\(Risk\_measure_{i,j} = \) various risk measures (e.g., CCFAR)
CPP with Conditional Cash Flow at Risk Constraints

- CPP with CCFAR

\[
\max_{x,z,CS,\alpha,\kappa_\omega} \left[ E[CS_{\Omega_T}] - \xi \cdot CCFAR \right]
\]

\[
CCFAR = \left( \alpha + \frac{1}{1 - \beta} \sum_{\omega \in \Omega_T} \kappa_\omega \right)
\]

\[
CCFAR \leq R
\]

\[
\kappa_\omega \geq p(\omega) \cdot (b(\omega_0) - CS_\omega - \alpha)
\]

\[
\kappa_\omega \geq 0
\]

\[\xi = \text{positive multiplier}\]

(sufficiently small so that the CCFAR part does not have a significant impact on portfolio selection)
CPP with a Risk Constraint Matrix

\[
\max_{x,z,CS,\alpha_{i,j},\kappa_{\omega_{i,j}}} \left[ E[CS_{\Omega_T}] - \left( \sum_{i=1}^{n} \sum_{j=1}^{m} \xi_{i,j} \cdot CCFAR_{i,j} \right) \right]
\]

where \( CCFAR_{i,j} = \left( \alpha_{i,j} + \frac{1}{1 - \beta_{j}} \sum_{\omega \in \Omega_{i}} \kappa_{\omega_{i,j}} \right) \),

\[
\kappa_{\omega} \geq p(\omega) \cdot (b(\omega_0) - CS_{\omega} - \alpha),
\]

\[
\kappa_{\omega} \geq 0
\]

subject to \( CCFAR_{i,j} \leq R_{t_i,\beta_j} \), \( M = \begin{pmatrix} R_{t_1,\beta_1} & R_{t_1,\beta_m} \\ \cdot & \cdot \\ R_{t_n,\beta_1} & R_{t_n,\beta_m} \end{pmatrix} \)

\( \xi_{i,j} \) = multipliers (sufficiently small values)
Experimental Setup (1/2)

- 5-year planning horizon with annual discretization
  - No short selling or funds borrowing
  - Assets:
    » Initial cash position $15 million
    » Total 60 (k=1,…  60) three-year projects (20 for years 0-3, 20 for years 1-4, 20 for year 2-5)
    » Industry specific stock index
    » Deposits at the risk free interest rate 2%

- Computational environment
  - 700MHz Pentium III processor, 256 MB of RAM, and Windows XP operating system
  - Dash Optimization software Xpress
Experimental Setup (2/2)

- **Industry specific stock index**
  - Scenario tree
  - Initial value, \( I(\omega_0) = 100 \)
  - Volatility, \( \sigma = 8\% \)
  - Logarithm of expected yearly growth rate \( \nu = 4\% \)
  - Cost at \( C_k \sim \$1 \text{ million} \times N(0,1) \)

- **Projects cash flows**
  - Initial cost generated randomly from a log normal distribution
  - Cash flows \( \Pi_k(q, \omega_l) \) for next three periods \( q = 1, 2, 3 \), when product lifecycle multipliers are \( plc(1) = 0.2, plc(2) = 0.5, plc(3) = 0.3 \)

\[
\Pi_k(q, \omega_l) \sim C_k \times plc(q) \times N \left( \frac{I(\omega_l) - I(\omega_0)}{I(\omega_0)} + 0.05, 0.2 \right)
\]
Mean-EDR Efficient Frontier

![Graph showing Mean-EDR Efficient Frontier](image-url)
Expected Cash Flow and CCFAR

Objective function:

$$\max \left( (1 - \lambda) \cdot E[C_{\Omega_T}] - \lambda \cdot EDR \right)$$

Risk constraint:

$$CCFAR_{5.95\%} \leq R$$
Expected Cash with 95% and 99% CCFAR Constraints
Expected Cash with 4 and 5 Year CCFAR Constraints
Some conclusions

- **Contributions**
  - Formulation of a Risk Constraint Matrix (RCM)
  - Introduction of conditional cash flow at risk (CCFAR) as a risk measure
  - Integration CCFAR into the CPP framework

- **Findings**
  - Mean-CCFAR model with RCM can reduce extreme risks significantly without appreciable losses in expected portfolio value
  - Importance of using concurrent risk constraints on different levels and intermediate periods
CPP in the Management of Electricity Contracts

- Unique characteristics of electricity markets (Bunn, 2004)
  - Non-storable, stakeholders bear price and load risk
  - High correlation between price and load
  - Mean reversion, spikes and seasonal variations
  - Volatility clustering, high and volatile risk premiums in futures

- How should an electricity generator/distributor hedge its risks using futures?
  - Scenario tree to account for correlation, arbitrage free, mean reversions, volatility clustering (Ho, et. al., 1995)
  - Risk requirements accounted for with CCFAR/CPP with a Risk Constraint Matrix (Kettunen and Salo, 2006)
Scenario Tree with Two Example Paths Highlighted

\[ S^t = \{ s^t \in \mathbb{R}_{2 \times t}^t | s^t_{i,j} \in \{0, 1\}, \quad i = P, L, \quad j = 1, ..., t \} \]

\[ s^1 = \begin{pmatrix} s^1_{P,1} \\ s^1_{L,1} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{pmatrix} \]

\[ s^2 = \begin{pmatrix} s^2_{P,1} \\ s^2_{L,1} \\ s^2_{P,2} \\ s^2_{L,2} \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{pmatrix} \]

Number of nodes = \( (t + 1)^2 \)

Number of scenarios = \( 4^t \)
Computational experiments

- Electricity distributor: uncertain load and price and can use futures to hedge risks
- Price data (€/MWh) from Nordpool 1999-2005 and futures seen on 24.3.2006
- Load data (GWh) from Finnish Energy Industries 1999-2005 (used 1% of actual)

<table>
<thead>
<tr>
<th>Delivery Period</th>
<th>Future</th>
<th>Price</th>
<th>Spot Volatility</th>
<th>Premium on Spot</th>
<th>Expected Load</th>
<th>Load Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.3-2.4</td>
<td>$M_0(0)$</td>
<td>54.69</td>
<td>$\sigma_P$</td>
<td>0</td>
<td>$\pi_0$</td>
<td>0.161</td>
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<tr>
<td>3.4-9.4</td>
<td>$M_1(1)$</td>
<td>54.40</td>
<td>$\sigma_P$</td>
<td>0.162</td>
<td>$\pi_1$</td>
<td>0.179</td>
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<td>10.4-16.4</td>
<td>$M_2(2)$</td>
<td>52.50</td>
<td>$\sigma_P$</td>
<td>0.199</td>
<td>$\pi_2$</td>
<td>0.199</td>
</tr>
<tr>
<td>17.4-23.4</td>
<td>$M_3(3)$</td>
<td>52.40</td>
<td>$\sigma_P$</td>
<td>0.211</td>
<td>$\pi_3$</td>
<td>0.216</td>
</tr>
<tr>
<td>24.4-30.4</td>
<td>$M_4(4)$</td>
<td>51.95</td>
<td>$\sigma_P$</td>
<td>0.219</td>
<td>$\pi_4$</td>
<td>0.234</td>
</tr>
<tr>
<td>1.5-7.5</td>
<td>$M_5(5)$</td>
<td>50.00</td>
<td>$\sigma_P$</td>
<td>0.232</td>
<td>$\pi_5$</td>
<td>0.253</td>
</tr>
</tbody>
</table>

- Conditional volatilities (fitting GARCH(1,1) for filtered data)
- Premiums (fitting linear equation)
- Mean reversions $c_P=0.2$ and $c_L=0.4$ (fitting linear equation)
- Correlation: $N=0.08$ and $\lambda=0.1$ (fitting linearized version of $\rho(s^t) = Ne^{\lambda L(s^t)}$)
- Risk free interest rate 2%
- Trade fee 0.03€/MWh
Contingent vs. periodic Optimization vs. fixed Allocation

5.6% cost reduction

Figures in million euros
Many other application areas

Characteristics
- Problems with numerous products / assets
- One or few shared uncertainties
- Examples
  » Electricity production / contract portfolio
  » Forest / farming / mining industry
  » Finance / real estate

Scenario trees
- Generation techniques (correlations, jumps, etc...)
- Scenario reduction techniques (maintain extreme events)