We consider efficient pricing of the electricity derivatives in competitive markets. In our approach both the price and the customer’s consumption are stochastic processes. With the consumption and price processes we formulate a money amount that the customer spends into electricity consumption. This money amount is the underlying asset of the fixed budget contracts. By using the fixed budget instruments the customer can budget his/her electricity consumption beforehand. Moreover, we derive an analytical solution for pricing these fixed budget contracts. The proposed method makes the pricing of deterministic standard load profiles extremely simple. Moreover, the proposed fixed budget instruments are an important addition to the contract portfolio of an energy retail company, since the ability to fill the customers needs is one of the keys to success in energy retail markets.

Keywords: Electricity pricing, stochastic consumption, competitive markets, tariff design
1. INTRODUCTION

Electricity supply markets have been at least partially opened for competition in nearly all European Countries after February 20\textsuperscript{th} 1999. Some of the EU countries have already opened all of their electricity retail market to competition, e.g., UK, Norway, Finland, and Sweden. The main reasons for this deregulation of electricity supply industry are increasing the customer value of the electricity service provided and also open the expensive and demanding electricity generation industry for private investments.

Efficient pricing of electricity services is the main strategic issue for electricity retail companies in the competitive electricity supply market. Based on experiments from the Scandinavian and UK markets, the total cost of service seems is the key factor for success. After a short transition period, customers seem to take the other issues like the quality of service and suppliers ability to meet the customer's individual needs as granted. Brands do neither have large importance in the energy commodity markets. Even if it is possible to make brands in energy sector, e.g., Green Energy, the customers are not willing to pay extra for the brand. They take the environmentally clean energy production as a self-evident part of the supplier's service. However, differentiated products will certainly represent market niches, with some firms offering high price high brand electricity services. See, e.g., Daneckere and Peck (1995) or Carlton (1978) for theoretical results of analysis of differentiated electricity products. In this paper, we define fixed budget instruments that allow the customers to fix the money amount that they are going to spend on electricity consumption during the supply contract period. Moreover, we derive pricing formulas for these fixed budget instruments in competitive electricity markets.

The price data of electricity contingent claims is needed in order to derive the process of the underlying asset and the pricing functions for fixed budget instruments. This data is obtained from organized exchanges where electricity spot, future products and also some electricity options are traded. These products are now traded on many different exchanges throughout the world, e.g., Nordpool (Scandinavia), APX (USA and Netherlands), NYNEX (New York) and VicPool (Victoria, Australia). Huge volumes of energy derivatives are also traded
over the counter by brokers, producers and suppliers. The customer’s consumption model is also needed in the
calculation of the underlying asset’s value. The large majority of residential and business customers are still
operating by using single suppliers. This electricity supplier or electricity retail company has normally some
historical or other background knowledge from the customer's electricity consumption. Having this specific
customer past consumption behavior, the supplier forecasts the customer's demand behavior over contract
period. The forecasts of customer's consumption process has normally high volatility since the consumption
forecast is normally needed to made at least a year ahead. For different analysis of consumption processes see,
e.g., Brown and Sibley (1986), Bunn and Farmer (1985), Chao (1983) and Räsänen, Ruusunen and Hämäläinen

Our main contribution is to apply the methodology from financial theory to the pricing of the fixed budget
electricity instruments. The pricing of electricity contingent claims is considered e.g. in Keppo and Räsänen
(1999). In this paper, we define the electricity fixed budget instruments and present a framework to price these
instruments in competitive markets. We formulate the customer's electricity budget and the corresponding
forward price process. The results obtained from our model show that the uncertainties in demand behavior must
be taken into account in the pricing process. If the standard load profiles without the consumption uncertainty
are used the only uncertainty concerning the electricity budget comes from the electricity price process.
Moreover, this paper shows that the more the customer’s load pattern uncertainty is correlated with the
electricity price uncertainty the more efficient the pricing is, since the correlated part of the consumption
uncertainty can be completely hedged in the electricity markets.

The paper is organizes as follows: Section 2 introduces the price and consumption models used in the paper.
The stochastic process for the money amount that the customer spends on consumption is derived in this section.
The processes are then applied in the pricing problem. Section 3 derives the pricing model and Section 4
illustrates the model with a simple example. Finally, Section 5 summarizes the main results of this paper.
2. ASSUMPTIONS AND MODELS

We consider an electricity market where energy instruments are traded continuously within a time horizon \([0, \tau]\). This kind of market exists in Scandinavian countries, where electricity producers and suppliers trade electricity 24 hours each day in a year either in the Nordpool-exchange or in over-the-counter markets. We consider single consumer that is defined by a consumption process \(Q(\cdot)\). For the description of the probabilistic structure of the markets, we will refer to an underlying probability space \((\Omega, F, P)\). Here \(\Omega\) is a set, \(F\) is a \(\sigma\)-algebra of subsets of \(\Omega\), and \(P\) is a probability measure on \(F\). The following assumptions characterize our electricity markets.

**ASSUMPTION 1:** The electricity price and the consumption processes follow the following Itô stochastic differential equations

\[
\begin{align*}
(1) \quad &dS(t) = S(t)\alpha_s(t)dt + S(t)\sigma_s(t)dB(t) \\
(2) \quad &dQ(t) = Q(t)\alpha_q(t)dt + Q(t)\sigma_q(t)dB(t)
\end{align*}
\]

where \(\alpha(t): [0, \tau] \to \mathbb{R}\) and \(\sigma(t): [0, \tau] \to \mathbb{R}\) are given functions that satisfy Lipschitz and growth conditions and \(B(\cdot)\) is a standard Brownian motion on the probability space \((\Omega, F, P)\), along with the standard filtration \(\{F_t: t \in [0, \tau]\}\).

Equation (1) says that the stochastic process for electricity price follows Itô process where the conditional expected change is \(S(t)\alpha_s(t)\) and \(S(t)^2\sigma_s(t)^2\) is the conditional variance of \(S(t)\). Assumption 1 guarantees the existence and uniqueness of the solution to (1) and (2), and it says that the uncertainty in the electricity market is generated from the Brownian motion process. Thus, we assume that the consumption and the price processes are perfectly correlated. This assumption is made in order to guarantee that the uncertainties in the consumption pattern have prices in the electricity markets. Usually, the process of the consumption is given as
\( dQ(t) = Q(t)\alpha(t)dt + Q(t)\sigma(t)dB(t) + Q(t)\tilde{\sigma}(t)d\tilde{B}(t) \)

where \( \tilde{B}(\cdot) \) is a Brownian motion that is independent of \( B(\cdot) \) and \( \tilde{\sigma}(\cdot) \) is the volatility process with respect to \( \tilde{B}(\cdot) \) (see, e.g., Chao 1983). Here, we consider only the first and the second component of equation (3).

The following assumption ensures that we can avoid pricing models that explicitly include market price of risk parameters.

**Assumption 2:** There exist future contracts on electricity price. Markets are complete and there is no arbitrage.

In a competitive electricity markets, there are huge amounts of future contracts continuously traded in exchange places, and this implies the electricity derivative markets have priced the risk associated with electricity price process. Given Assumption 2 there are no opportunities for risk-less profits in the market. If there exists such risk-less trading opportunities, the traders and customers will collect those instruments out from the market (see, e.g., Brown and Sibley, 1986). Market's completeness means that the tradable instruments are priced according to the same unique linear pricing function [see e.g. Duffie (1992)]. Under Assumption 2 the price of a \( T \)-maturity instrument at time \( t \) is given as

\[
F(t,S) = E[\pi(T)F(T,S) | F_t] \quad \text{for all} \quad t \in [0,T], \ T \in [0,\tau]
\]

where \( E \) is the expectation operator, \( \pi(T) = \exp\left\{-\int_t^T \left[ r(s) + \frac{1}{2} \lambda(s)^2 \right] ds - \int_t^T \lambda(s) dB(s) \right\} \), \( r(\cdot) \) is the risk free interest rate process, and \( \lambda(\cdot) \) is the market price of risk process that is implicitly given by (4) because \( F(t,S) \) is got from the markets.

The value of the consumption pattern means the amount of money that the customer spends on the consumption of the electricity at time \( t \). That is, the value of the customer’s consumption pattern

\[
P(t) = Q(t)S(t) \quad \text{for all} \quad t \in [0,\tau]
\]
In this paper, we only consider the derivative pricing on the part of the money amount that perfectly correlates with electricity price [i.e., the first terms in (3)]. We do not define any contingent claims on the other part of the consumption pattern. The instruments that depend on the uncorrelated part of the consumption pattern cannot be hedged in electricity markets. Therefore, the delivery of the uncorrelated part of the money amount can be priced, e.g., by $1.1 \cdot S(t) \left[ \hat{Q}(t) - Q(t) \right]$, where $\hat{Q}(t)$ is a solution to (3). That is, because the uncorrelated part is also delivered to the customer, it can be priced by using the market price of the money amount plus a reasonable risk margin, which in Scandinavian market can be for example, 10 percent margin.

**Lemma 1:** The process for the value of the consumption pattern is

\[
dP(t) = P(t)\left[ \alpha_s(t) + \alpha_{\hat{Q}}(t) + \sigma_s(t)\sigma_{\hat{Q}}(t) \right]dt + P(t)\left[ \sigma_s(t) + \sigma_{\hat{Q}}(t) \right]dB(t)
\]

for all $t \in [0, \tau]$

**Proof:** Using Itô’s lemma, see, e.g., Øksendal (1995), and equation (5) we get (6). Q.E.D.

From Lemma 1 we get

\[
P(T) = P(t) \exp \left[ \int_t^T \left[ \alpha_s(y) + \alpha_{\hat{Q}}(y) + \sigma_s(y)\sigma_{\hat{Q}}(y) - \frac{1}{2} \left[ \sigma_s(y) + \sigma_{\hat{Q}}(y) \right]^2 \right] dy 
\]

\[
+ \int_t^T \left[ \sigma_s(y) + \sigma_{\hat{Q}}(y) \right] dB(y) \right] \right]
\]

\[
(7)
\]

3. Pricing

Here, we consider the valuation of fixed budget contingent claims for a single customer’s consumption pattern. The presentation will be brief, since detailed analysis of contingent claims valuation can be found, e.g., Heath, Jarrow, and Morton (1992), Harrison and Kreps (1979), and Harrison and Pliska (1981). From assumptions 1 and 2, equation (4), and Lemma 1 we get
(8) \[ F(t, P) = E\left[ \pi(T)F(T, P) \mid F_t \right] \text{ for all } t \in [0, T], \ T \in [0, \tau] \]

That is, the contingent claims on \( P \) are priced using the pricing function of electricity contingent claims because the uncertainty in \( P \) is perfectly correlated with \( S \). If equation (8) does not hold there exists arbitrage opportunities by trading fixed budget instruments and electricity derivatives.

Employing equation (8) to forward contracts, i.e., now we assume that \( F \) is a forward contract, we get the following theorem.

**THEOREM 1:** *The time \( t \) \( T \)-maturity arbitrage-free forward price for \( P \) is given by*

\[ P(t, T) = \frac{E\left[ \pi(T)P(T) \mid F_t \right]}{E\left[ \pi(T) \mid F_t \right]} \text{ for all } t \in [0, T], \ T \in [0, \tau] \]

**PROOF:** Equation (8) gives (10), because \( F(t, P) \) is now zero. \( Q.E.D. \)

Theorem 1 is a pricing rule for \( T \)-maturity forward price on \( P \). The value of the forward contract when initiated is by definition zero. By allowing \( T \) to vary from \( t \) to \( \tau \) Theorem 1 solves the whole forward price curve.

As noted above, all fixed budget contingent claims can be priced by using equation (8). Given Theorem 1 we get the following pricing rule for call options on \( P \).

**THEOREM 2:** *If the risk-free rate is deterministic, the time \( t \) \( T \)-maturity arbitrage-free call option price on \( P \) is given by*

\[ c(t, T) = \exp \left[ -\int_t^T r(s)ds \right] \left[ P(t, T)N(d_1) - K(T)N(d_2) \right] \text{ for all } t \in [0, T], \ T \in [0, \tau] \]

where \( K(T) \) is the strike price for the \( T \)-maturity option, \( N(\cdot) \) is cumulative normal distribution,
\[
\begin{align*}
    d_1 &= \frac{\ln[P(t,T)/K(T)] + \frac{1}{2} \int_t^T [\sigma_S(y) + \sigma_Q(y)]^2 dy}{\sqrt{\int_t^T [\sigma_S(y) + \sigma_Q(y)]^2 dy}} \\
    d_2 &= d_1 - \sqrt{\int_t^T [\sigma_S(y) + \sigma_Q(y)]^2 dy}
\end{align*}
\]

**PROOF:** By using the model of Black (1976), Theorem 1, and equation (8) we get (11). \(Q.E.D.\)

The corresponding put option can be solved in the same way. Typically tariffs are understood as a portfolio of different maturity call options each of which are priced according to Theorem 2. That is, from (11) we get that the value of customer's fixed budget tariff is

\[(12)\quad C(t,T) = \int_t^T c(t,y) dy \quad \text{for all} \quad t \in [0,T], \ T \in [0,\tau]\]

### 4. EXAMPLE

We illustrate our fixed budget electricity pricing model with an example. We consider pricing of a large industrial customer's electricity contract. The pricing period is July 1\(^{st}\) 1999 – May 31\(^{st}\) 2000. The current date is April 1\(^{st}\) 1999 and the time interval is divided into one hour pricing periods. The electricity market price process is observed from the Scandinavian Electricity Exchange Nordpool. For simplicity, we assume that \(r = 0\) and \(\lambda = 0\).

The expected electricity price is shown in Figure 1. The mean volatility on the time period is 0.03.
The corresponding consumption pattern of the customer is illustrated in Figure 2. The mean volatility during the time period is equal to 0.01. The uncertainties between hourly price and consumption process are assumed to be perfectly correlated.
From figures 1 and 2, Lemma 1, and Theorem 1 we get the forward price curve for the money amount that the customer spends on electricity consumption. Figure 3 illustrates the forward price curve.

![The hourly forward price curve of the consumption money amount](image)

Figure 3. The hourly forward price curve of the consumption money amount

Figure 3 defines hourly money amounts that the consumer has, when he/she has the long forward contracts. Next, we calculate the value of the fixed budget contract given by Theorem 2 and equation (12). To illustrate our approach, we consider two different strike structures. The other tariff has a constant strike price that is equal to 4.18 FIM. The strike structure of the other tariff is illustrated in Figure 4. The option price of the fixed budget tariff with the constant strike price is 12 684 FIM and the option price of the other tariff is 9 133 FIM. Thus, these are the option prices that give the customer an opportunity to pay his/her electricity consumption according to the strike price structures. With both these tariffs the cumulative strike price is 36 717 FIM. The option value of the constant tariff is higher because the expected profit from the contract is higher than from the latter tariff.
5. CONCLUSIONS

In this paper we have derived an approach to price fixed energy budget instruments in competitive electricity markets. The underlying asset of these instruments is the money amount that the consumer spends on electricity consumption. The pricing of fixed budget instruments is based on electricity market data obtained from electricity exchange. The result shows that more there is uncertainty either in price or consumption, the higher the fixed charge of fixed budget tariff. Moreover, we have shown that the best kind of uncertainty in the consumption process is the one that is perfectly correlated with electricity spot price. This perfectly correlated uncertainty in consumption can be hedged by using the electricity derivatives traded in the exchanges. The proposed approach can also be applied for hedging of standard load curve based electricity contracts which are applied in Scandinavian and UK electricity retail markets, since the correlation between the standard load curves and spot price is known.
References


