An Interactive Evolutionary Multi-Objective Optimization Method Based on Progressively Approximated Value Functions

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Outline

➢ Introduction
➢ Past Studies
➢ PI-EMO
➢ Results
➢ Parametric Study
➢ Results - DM with Random Error
Introduction

• Approaches to handle many objective optimization problem
  • Aposteriori Approach
  • Apriori Approach
  • Semi-interactive Approach
  • Progressively Interactive Approach
• A Progressively Interactive EMO Approach (PI-EMO) has been suggested
• Preference information used to construct a Value function
• Results have been presented on 2 to 5 objectives test problems
• Inconsistency in decision-maker responses has been studied
Past Studies

- Phelps and Koksalan optimized the constructed linearly weighted sum of objectives in subsequent iterations using an evolutionary algorithm.
- Fowler et al. developed an interactive EMO approach based on the idea of using convex preference cones.
- Jaszkiewicz selected a set of linear value functions (based on weighted sum of objectives) from a set of randomly created linear value functions, conforming to the preference information supplied by the DM.
- Branke et al. implemented the GRIP methodology in which the DM compares pairs of alternatives and the preference information thus obtained is used to find all possible compatible additive value functions.
- Korhonen, Moskowitz and Wallenius suggested a progressive and interactive multi-objective optimization and decision-making algorithm in which the DM is presented with a set of alternatives and is asked to make a set of binary comparisons (motivation).
PI-EMO

Value Function for Two-Objectives

\[ V(f_1, f_2) = (f_1 + k_1 f_2 + l_1)(f_2 + k_2 f_1 + l_2), \]

where \( f_1, f_2 \) are the objective functions and \( k_1, k_2, l_1, l_2 \) are the value function parameters.

Higher objective value functions have more number of product terms.
\[(f_1 + k_1 f_2 + l_1)(f_2 + k_2 f_1 + l_2) = c\]

Proposed Value Function

Value Function from Optimization
Results on Two-Objective Modified ZDT1

Maximize \( f(x) = \left\{ \frac{x_1}{10 - \sqrt{x_1 g(x)}} \right\}, \)

where \( g(x) = 1 + \frac{9}{29} \sum_{i=2}^{30} x_i, \)

\( 0 \leq x_i \leq 1, \quad \text{for} \ i = 1, 2, \ldots, 30, \)

Value Function

\[ V(f_1, f_2) = \frac{1}{(f_1 - 0.35)^2 + (f_2 - 9.6)^2}. \]

Most Preferred Point

\[ z^* = (0.25, 9.50). \]
Results on Two-Objective Modified ZDT1

Evolution of value functions after successive DM calls.

Value functions near the most preferred point.
Results on Three-Objective Modified DTLZ2

Maximize  \( f(x) = \begin{cases} 
(1.0 + g(x)) \cos(\frac{\pi}{2} x_1) \cos(\frac{\pi}{2} x_2) \cdots \cos(\frac{\pi}{2} x_{M-1}) \\
(1.0 + g(x)) \cos(\frac{\pi}{2} x_1) \cos(\frac{\pi}{2} x_2) \cdots \sin(\frac{\pi}{2} x_{M-1}) \\
\vdots \\
(1.0 + g(x)) \cos(\frac{\pi}{2} x_1) \sin(\frac{\pi}{2} x_2) \\
(1.0 + g(x)) \sin(\frac{\pi}{2} x_1) 
\end{cases} \)

subject to  \( 0 \leq x_i \leq 1, \) for \( i = 1, \ldots, 12, \)

where  \( g(x) = \sum_{i=3}^{12} (x_i - 0.5)^2. \)

\[ V(f_1, f_2, f_3) = 1.25f_1 + 1.50f_2 + 2.9047f_3. \]

Most Preferred Point

\[ z^* = (1.2500, 1.5000, 2.9047) \]
Results on 3-Objective Modified DTLZ 2 (High Accuracy)

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
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</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>0.0008</td>
<td>0.0115</td>
<td>0.0434</td>
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<td>Func. Evals.</td>
<td>4,200</td>
<td>6,222</td>
<td>8,982</td>
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<td># of DM Calls</td>
<td>17</td>
<td>25</td>
<td>36</td>
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Results on 5-Objective Modified DTLZ 2 (High Accuracy)

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<tbody>
<tr>
<td>Accuracy</td>
<td>0.0084</td>
<td>0.0240</td>
<td>0.0902</td>
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<td># of Function Eval.</td>
<td>23,126</td>
<td>27,202</td>
<td>41,871</td>
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<tr>
<td># of DM Calls</td>
<td>57</td>
<td>67</td>
<td>102</td>
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</table>
DM with Random Error

\[ V(f_1, f_2, f_3) = \text{noise}(1.25, \sigma)f_1 + \text{noise}(1.50, \sigma)f_2 \\
+ \text{noise}(2.9047, \sigma)f_3, \]

Final solutions obtained by PI-NSGA-II for the three-objective modified DTLZ2 problem with a stochastic DM-emulated value function.

<table>
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<tr>
<th>( z^* )</th>
<th>Best</th>
<th>Median</th>
<th>Worst</th>
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<tbody>
<tr>
<td>1.2500</td>
<td>1.2555</td>
<td>1.2695</td>
<td>1.2902</td>
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<tr>
<td>1.5000</td>
<td>1.5105</td>
<td>1.5205</td>
<td>1.6437</td>
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<tr>
<td>2.9047</td>
<td>2.8969</td>
<td>2.8856</td>
<td>2.8078</td>
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</table>

DM emulated with a noisy Value Function on the Three-Objective DTLZ2 Test Problem

Distance of obtained solution from the most preferred solution, function evaluations, and the number of DM calls by PI-NSGA-II for the three-objective modified DTLZ2 problem with a stochastic DM-emulated value function.

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>0.0142</td>
<td>0.0342</td>
<td>0.1779</td>
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<tr>
<td>Func. Evals.</td>
<td>5,841</td>
<td>7,608</td>
<td>9,663</td>
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<tr>
<td># of DM Calls</td>
<td>24</td>
<td>31</td>
<td>39</td>
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</table>
Thank You