Patent Valuation using Fuzzy Numbers

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Patent Valuation

Patent valuation is always a difficult task due to the nature of intellectual property:

- There could be great uncertainty with regard to both the technical and commercial success in competitive markets of the underlying technology.
- There could be uncertainties about the legal challenges which can occur both during the application and subsequent enforcement.
- There is significant inaccuracy in our ability to estimate future added cash-flows that our patents will generate.

Fuzzy pay-off method for real option valuation

Two recent papers [1, 2] present a novel approach to real option valuation, namely the Datar-Mathews method, which suggests to consider the real option value as the probability-weighted average of the pay-off distribution, where the negative outcomes of project imply a zero pay-off. The Fuzzy Pay-Off method [3] uses fuzzy numbers in representing the profitability (NPV) outcomes, which calculates the real option value by

$$ROV = \frac{\int_0^\infty A(x)dx}{\int_{-\infty}^\infty A(x)dx} \times E(A_+), \qquad (1)$$

where A is the fuzzy NPV and $E(A_+)$ stands for the fuzzy mean value of the positive side of the NPV.

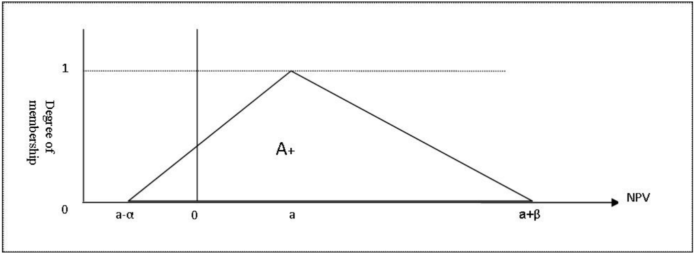

Figure: An example of NPV distribution which is treated as a triangular fuzzy number.

$\int_{-\infty}^{\infty} A(x)dx$ computes the area below the whole NPV distribution A and $\int_{0}^{\infty} A(x)dx$ computes the area below the positive part of A. Therefore, equation (1) suggests that real option value is nothing more than the weighted fuzzy mean of the positive values of the fuzzy NPV.

Possibilistic mean value and variance of fuzzy numbers

In 2001 Professor Christer Carlsson and Professor Robert Fuller [4] introduced the notions of possibilistic mean value and variance of fuzzy numbers. Afterwards they also explained and illustrated these notions in a pure probabilistic aspect of view in [5].

They defined the f-weighted possibilistic mean value of fuzzy number A as

$$E_f(A) = \int_0^1 E(U_\gamma) f(\gamma)\, d\gamma = \int_0^1 \frac{a_1(\gamma) + a_2(\gamma)}{2} f(\gamma)\, d\gamma,$$

and the f-weighted possibilistic variance of A has been defined as

$$Var_f(A) = \int_0^1 \sigma^2_{U_\gamma} f(\gamma)\, d\gamma = \int_0^1 \frac{(a_2(\gamma) - a_1(\gamma))^2}{12} f(\gamma)\, d\gamma,$$

where U_γ is a uniform probability distribution on $[A]^\gamma$ for all $\gamma \in [0, 1]$.

The f-weighted measure of possibilistic covariance between fuzzy numbers A and B (with respect to their joint distribution C) is given as

$$Cov_f(A,B) = \int_0^1 Cov(X_\gamma, Y_\gamma) f(\gamma)\, d\gamma,$$

where X_γ and Y_γ are random variables such that their joint distribution is uniform on $[C]^\gamma$for all $\gamma \in [0,1]$.
The function $f : [0,1] \to \Re$ in the above formulas is the weighting function, which satisfies the following conditions:

- f is non-negative and monotone increasing.
- $\int_0^1 f(\gamma)\, d\gamma = 1$.

- V. Datar and S. Mathews, A practical method for valuing real options: the boeing approach, Journal of Applied Corporate Finance, vol. 19, no. 2, pp. 95–104, 2007.
- S. Mathews and J. Salmon, Business engineering: a practical approach to valuing high-risk, high-return projects using real options, in Tutorials in Operations Research, P. Gray, Ed., Informs, Hanover, Md, USA, 2007.
- Mikael Collan, Robert Fullér and József Mezei, A Fuzzy Pay-Off Method for Real Option Valuation, Journal of Applied Mathematics and Decision Sciences Volume 2009 (2009), Article ID 238196, 14 pages doi:10.1155/2009/238196
- Christer Carlsson and Robert Fuller, On possibilistic mean value and variance of fuzzy numbers, Fuzzy Sets and Systems, 122(2001) 315-326
- Christer Carlsson and Robert Fuller, Possibilistic Mean Value and Variance of Fuzzy Numbers: Some Examples of Application, FUZZ-IEEE 2009, Korea, August 20-24, 2009